The Behavior of Beta in the 19th Century

Lord Mensah[1]

[1] University of Ghana Business School, Department of Finance, Legon, Accra-Ghana

Correspondence: Lord Mensah, University of Ghana Business School, Department of Finance, Legon, P. O. Box 78, Legon, Accra-Ghana. E-mail: lordmensah@ug.edu.gh

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Abstract

This paper uses completely new data to study the variations in beta when it deviates from the constancy assumption presumed by the market model. The concentration of the literature on beta is based on post 1926 data. This makes the 19th century Brussels Stock Exchange (BSE) data a very good out-sample dataset to test beta variations. Various models proposed in the literature to capture the variations in beta were studied. Blume's correlation techniques reveal that beta is not stable at the individual stocks level and that the stability can be fairly improved by portfolio formations. Using root mean square error (RMSE) criterion, it was shown that the market model betas are weak in predicting future betas. The predictability can be improved by adjusting betas with the Blume and Vasicek mean reversion techniques. Further results from this study reveal that few stocks have lead or lag relationship with the market index. Small sized stocks were detected to be more prone to outliers. In effect betas in the 19th century exhibits similar pattern as betas in the post 1926 era.

Keywords: Financial history, Historical beta estimation, Brussels stock exchange, Robust betas

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1. Introduction

Beta stability, bias and robustness evaluations have become the centre stage of research in finance since the development of the Capital Asset Pricing Model (CAPM) by Sharpe (1964). Beta, one of the parameters of the time series regression, plays a key role in CAPM applications. Beta is usually estimated with the standard market model (MM). The MM is a statistical model that relates the return of any stock to the return of the market index. Beta estimated with the MM has been tested not to be stable over time. The financial literature has indicated that the stability can be improved by Blume's and Vasicek's autoregressive adjustment methods (Blume (1971), Vasicek (1973)). The objectives of these adjustment methods are based on the tendency of betas from successive periods to be mean reverting. While Vasicek considers the accuracy of the historical betas, Blume depend on how far the historical betas deviate from their average value. The literature has also shown that betas estimated by the market model are biased. That is when stocks are ranked based on their market capital (size); beta is biased downwards for small size stocks and upwards for large size stocks. Scholes and Williams (1977), Dimson (1979), Fowler and Rorke (1983) and Marsh (1979) developed methods to correct the bias. The method adopted by Marsh (1979) is MM estimation with returns calculated on trade to trade basis. Dimsom (1979) adopted the aggregate coefficient method which computes the lead and the lag betas in a multiple regression of the stock return on a number of lead and lag market returns. The sum of the estimated beta coefficients from the multiple regressions is the Dimson's estimate of beta. On the other hand, Scholes and Williams (1977) lead and lag beta coefficients are computed from univariates regressions. The literature also highlights the impact of outliers (extreme observations) on the beta estimates. It proposes various techniques of minimizing the impact of outliers on beta estimates. The weighted least square estimation technique is one of the methods used in minimizing the impact of outliers on beta estimates. Based on the assumption that outliers have lower probability of occurring in the future, less weight is placed on them (Martin et al. (2003)). An alternative technique is to get rid of the extreme observations and perform the regression with fewer observations.

To the best of my knowledge the financial literature has not traced the stability, bias and robustness of beta using data beyond 1926 in any country. Luoma et al. (1994) generated different artificial market returns data and documented that different beta estimation techniques behave differently on different markets. This shows that there is no beta adjustment technique which has been accepted universally and performs well independent of the data.

The objective of this study is to introduce a new set of data from the 19th Century Brussels Stock Exchange (BSE) to test the performance of the alternative techniques of estimating beta. By so doing one would know the beta estimation technique needed to estimate beta on this data. Individual and portfolio betas from the various estimation techniques will be compared based on their predictive accuracy. The root mean square error (RMSE) and the mean absolute error (MAE) criterion adopted by Blume (1971) and Klemkosky et al. (1975) are used as an error predictor of the various techniques. The 19th century data provide a very good platform to determine if betas before 1914 exhibit similar pattern as betas after 1914.

This paper is organized as follows: the next section provides a summary of related literature on stationarity, biased and robustness of beta. In section 3, various models of estimating beta are discussed. Section 4 provides a detailed description of the data and descriptive statistics of beta when estimated with the least square regressions of the market model. In section 5, Blume's correlation technique of detecting the stability of beta is studied. Betas are adjusted with the Blume and Vasicek's autoregressive techniques in section 6. The predictive accuracy of the adjustment techniques based on the root mean square error criterion is tested in the same section. The modified Diebold and Mariano test proposed by Harvey et al. (1997) is used to test for equal predictive accuracy of the models. Section 7 determines how the stocks in each period lead or lag the market index using the Dimson's model. Betas estimated with the iterative reweighted least square (outlier resistant) techniques are compared to the benchmark market models betas in section 8. The final section presents the conclusion.

2. Related Papers

There is substantial evidence in the financial literature that has established that security betas are not stable over time. Particularly, previous studies have indicated that portfolio betas are more stable as compared with individual security betas. Blume (1971 and 1975) pioneered the research on beta stability focusing on the cross-sectional correlation between beta estimates for successive periods. His study is based on the assumption that beta is stationary within each period. He concluded that beta instability can be attenuated by the formation of portfolios. Subsequent publications on beta stability assume that beta variations exhibit stochastic behavior. They formally propose a model to capture the variations in beta overtime and conduct a test to confirm the model. For instance, Fabozzi and Francis (1978) found strong evidence of beta following the Hildreth-Houck random coefficient model. Collin et al. (1987) utilized Ohlsen and Rosenberg's model to analyze the variations in beta for both individual and randomly formed portfolios. Their findings were in support of the mixed sequential and purely random variations of the beta. It was clear from their results that individual securities and portfolio betas exhibit the mixed sequential and purely random variations. Faff et al. (1992) used Australian data to establish evidence in beta nonstationarity. Details of their result indicates a relationship between beta nonstationarity, the market index as a proxy for the general market movement, the magnitude of the betas (degree of riskiness) and the market capital. Further investigation by Gregory-Allen et al. (1994) confirm that when the variance of the beta estimate is taking into consideration, there is no evidence that portfolio betas are more stable than individual security betas (as claimed by Blume (1971)).

The focus of the above literature is on the stability of beta, but the issue of beta bias is worth consideration. The bias in beta estimates is traced in the literature to be due to infrequent trading. Ibbotson et al. (1997) documented that beta estimates for small stocks are severely downwards biased that is they exhibit severe infrequent trading. They recommend the inclusion of lagged market returns in the estimation of beta when securities are infrequently traded. On the other hand Bartholdy and Riding (1994) found on the New Zealand Market, the MM betas are less biased, more efficient and consistent compared to the Dimson's, Scholes and Williams betas (corrected for bias). In effect, the literature has shown that nonsynchronous trading biases betas estimated with the MM. When stocks are ranked based on their market capital (size), beta is biased downward for small size stocks and upward for large size stocks. Scholes and Williams (1977), Dimson (1979) and Fowler and Rorke (1983) developed methods to correct the bias. Dimson (1979) adopted the aggregate coefficient method, which computes the lead and the lag betas in a multiple regression of the stock return on a number of lead/lag market returns. The sum of the estimated beta coefficients from the multiple regressions is Dimson's estimate of beta.

In the MM least square estimation, it is assumed that the return residuals follow a normal distribution, and that influential points are rare. Practically this assumption fails most of the time. The failure is connected to the occurrence of a small fraction of exceptionally large or small returns (outlier or influential points) observation. Chatterjee and Jacques (1994) studied the effect of these points on the beta parameter. They established that the market model beta estimation method shield the outliers and aggregate their influence on the beta due to the square of their errors. They proposed that the effect of these observations on beta can be reduced by applying least square method that can detect and assign zero weights to the outlying observations in the estimation process. Chan and

Lakonishok (1992) emphasized the robust method (outlier detecting methods) of estimating beta are good for stocks which are susceptible to outliers (mostly due to stock splits, dividend cuttings, and initial public offerings (IPO's)). Stocks which exhibit this behavior are mostly small sized stocks. Martin and Simin (2003) confirmed the outlier resistance beta better predicts future beta than the market model beta. They also reveal that, small sized stocks betas are more vulnerable to outliers.

3. Various models for estimating beta

Firstly, beta will be estimated from the market model (MM) which is basically the statistical model that relates the return of any given stock to the return of the market index. Thus for returns of a stock j in period p we write:

$$R_{jp} = \alpha_j + \beta_j R_{mp} + \varepsilon_{jp}, \tag{1}$$

Where (Note 1) $E(\varepsilon_{jp}) = 0$ and $Var(\varepsilon_{jp}) = \sigma_{\varepsilon j}$, R_{jp} and R_{mp} are the period p returns on the stock j and the market index return respectively. α_j, β_j and $\sigma_{\varepsilon j}^2$ are the estimated parameters of the market model peculiar to stock j. The MM assumes that the estimated parameter β_j is constant over time but empirical evidence confirm otherwise. One has to establish a model to capture the variability of the beta. Beta coefficients estimated in successive periods have been shown to be mean reverting by Blume (1971). Blume uses this autoregressive tendency to improve the accuracy of beta forecasts. The estimation procedure adopted by Blume is obtained from cross-sectional regression of the betas of period p on the betas of period $p-1$. The betas of period $p-1$ are then substituted for period p betas to predict period $p+1$ beta coefficients.

$$\beta_{j,p} = a + b\beta_{j,p-1} \quad \text{for} \quad j = 1,2,\ldots,N, \tag{2}$$

where N is the number of stocks in the period.

Vasicek (1973) also utilized the autoregressive tendency of beta in a successive period to improve the betas in a period based on prior information on betas. Vasicek (1973) applied the Bayesian correction method by utilizing the cross-sectional information of the previous period betas:

$$\beta_{j,p} = \frac{\dfrac{\overline{\beta}_{j,p-1}}{\operatorname{var}(\beta_{j,p-1})} + \dfrac{\beta_{j,p-1}}{\sigma_\beta^2}}{\dfrac{1}{\operatorname{var}(\beta_{j,p-1})} + \dfrac{1}{\sigma_\beta^2}} \quad \text{for} \quad j = 1,2,\ldots,N, \tag{3}$$

where $\beta_{j,p}$ is the mean of the posterior distribution of beta for stock j which serves as the beta forecast. σ_β^2 is the variance of the market model regression coefficients $\beta_{j,p-1}$. $\overline{\beta}_{j,p-1}$ is the cross-sectional mean of betas in period $p-1$, and $\operatorname{var}(\beta_{j,p-1})$ is the variance in the cross-section of betas. Blume and the Vasicek Bayessian adjustment techniques above control the stability of beta.

Unbiased beta can be obtained by adopting the biased-correction procedures proposed by Dimson (1979). The Dimson's technique involves estimating a multiple regression of the form

$$R_{jp} = \alpha_j + \sum_{i=-3}^{3} \beta_j R_{m,p+i} + \varepsilon_{jp}, \tag{4}$$

The Dimson beta estimate is then given by $\beta_{\dim} = \sum_{i=-3}^{3} \beta_i$. Here R_{jp} is the return series of stock j and R_{mp} is the return series of the market index. The error term ε_{jp} is assumed to follow the assumptions of the classical linear regression model. More lag and lead terms will be required for very thinly traded stocks to correct for bias. Fowler and Rorke (1983) presented analytical evidence that the Dimson and Scholes-Williams methods do not sufficiently controls thin trading bias in beta estimation. They proposed a variant version of Scholes and Williams model which yields betas which are consistent and unbiased. We use only three months lag and lead because beta bias has been documented as not prevalent in monthly returns data (see Cohen, Hawawini, Maier, Schwartz and Whitcomb (1983)).

The least square approach adopted for the market model minimizes the sum of square of residuals with respect to the model parameters α_j and β_j:

$$\min_{\alpha_j,\beta_j} \left(\alpha_j - R_j - \beta_j R_{mp}\right)^2 .$$

Squaring the residuals magnifies the effect of the outliers on the estimated parameters. For instance, expected returns are likely to be shifted towards the outliers whereas covariance matrix will be inflated towards outliers. To reduce the influence of outliers, the statistics literature has emphasized on the use of iterative reweighted least-square (IRLS) method. This method estimates beta by iteratively minimizing a weighted function which depends on standardized return residuals:

$$\min_{\alpha_j,\beta_j} W\left(\frac{R_j - \alpha_j - \beta_j R_{mp}}{\sigma_{\varepsilon j}}\right) , \tag{5}$$

with weight function W and $\sigma_{\varepsilon j}$ the standard deviation of the return residual. First, estimate regression parameters from least square regressions and use these parameters as initial input for the iteration. A weight function is applied to the standardized residual. The bisquare weight function define as:

$W(s) = \begin{cases} \left[1 - \left(s/T\right)^2\right]^2 & for \quad |s| \le T \\ 0 & for \quad |s| > T \end{cases}$ with the standardized residual $s = \dfrac{R_j - \alpha_j - B_j R_{mp}}{\sigma_{\varepsilon j}}$ and T, a tuning constant is considered in this case. Small values of T introduce more resistance to outliers, but at the expense of efficiency when return residuals are Gaussian. Statistically, the value of T is chosen in order for the method to be 95- percent efficient when errors are normal and it will still provide protection against outliers. For the bisquare weight function the value of T is set to 4.685. With the initial input from the first least square regressions, standardized the return residual and use the weight function define above to transform them. Estimate the parameters by the weighted least square: $b^1 = \left[\Re' W^0 \Re\right]^{-1} \Re' W^0 R_j$ where $\Re = \left[\iota \quad R_{mp}\right]$ with ι a vector of ones with the same size as R_{mp} (market index returns). W^0 is an initial standardized weight diagonal matrix, b^1 the estimated parameter. The new set of parameters serves as input for the next iteration. The above procedure is repeated until the parameter of interest (beta) converges. With the above weight function, return pair observations whose absolute standardized residual exceeds the tuning constant is assigned zero weight (outlier).

4. The Data and Beta Coefficient Descriptive Statistics

The data considered for this study was taken from the Brussels Stock Exchange (BSE). The data set was constructed at the University of Antwerp in Belgium (Studiecentrum voor Onderneming en Beurs (SCOB)). The data for our study start from 1832 and ends at 1914, just before the outbreak of World War I. During the World War I period, the BSE was closed and this can be regarded as a natural breaking point of the long time series of the stock returns. The BSE was considered one of the biggest markets in the world at that time, because Belgium was one of the first nations on the European continent to become industrialized (see Van der Wee (1996) and Neymarck (1911)). On the industrial output per head ladder, Belgium stood second after Britain in 1860, and third in 1913, after the UK and the USA (see Bairoch (1982)). During this period, highly developed banking system coupled with liberal stock market regulations attracted a great deal of domestic and foreign capital in Belgium. In confirmation, Van Nieuwerburgh, Buelens and Cuyvers (2006) document that the development of the financial sector, accompanied with the stock market-based financing of firms, played an important role in the economic growth of 19th century Belgium. The database contains monthly common stocks return from 1832 to 1914 for every stock listed on the BSE as of that time. The returns have been adjusted for dividends and stocks split. Considering the environment the BSE was operating, the period studied can be divided into two: thus the period of high regulatory environment (1837-1866) and the period of deregulation and expansion (1867-1911). Nowadays, the BSE is not recognized as one of the top stock exchange but it is ranked among the largest stock markets in the world during the 19^{th} and the first half of the 20^{th} centuries (Annaert et al. (2004)). Therefore, studying the return patterns on this market can be compared to the current UK market. The stocks on the BSE are well diversified across industries such as transportation, mining and extraction, financials, utilities and industrials. The BSE as of that time listed Belgium and foreign owned stocks. This study considers only Belgium owned stocks.

The data sample is divided into fifteen five-year non-overlapping sub periods p_i for $i = 1,...,15$. Stocks with complete return data for a period of five years are considered in each sub period. The removal of stocks that did not trade fully within the five-year estimation periods does not introduce survivorship bias since there is no significant difference in the beta coefficient descriptive statistics when stocks with at least 24 observations are added to the sample (not reported). For the proxy of the market portfolio we consider value-weighted market index constructed by the studiecentrum. The number of stocks in each period and the statistical characteristics of the betas estimated in each period are presented in Table 1. The Table displays the fifteen periods studied starting from January 1837 to December 1911. We ignore the first five years since the period can only boast of only one listed stock. The number of stocks with full return data in each period ranges from 21 to 424 as shown in column 2. It can be seen in Table 1 that the number of stocks that have five years of data does not exceed 100 before 1877. The beta coefficient of each stock in each period is estimated by simply regressing the monthly returns of the stock on the corresponding value weighted index using model 1 above. Cross-sectional statistics of the betas in each period are computed and the result is displayed. Equal and value weighted cross-sectional average of betas are displayed in column 3 and 4 respectively. We compute the value-weighted mean beta by considering individual stock relative to market capitalization at the beginning of the period. One can easily see from the weighted average that small size stocks contribute relatively high beta values in almost every period.

Table 1. Beta Coefficient Descriptive Statistics for the 15 estimated periods

Period	Number of Stocks	Mean(EQ)	Mean(VW)	Standard Deviation	Percentage of BETAS <Zero	Minimun	Fractiles 0.10	0.25	0.50	0.90	Maximum	Mean R^2(%)
1/1837-12/1841	21	1.37	1.24	0.67	0	0.38	0.50	0.77	1.35	2.29	2.90	29
1/1842-12/1846	33	1.10	1.01	1.84	12	-1.18	-0.04	0.27	0.79	2.21	10.19	18
1/1847-12/1851	32	0.69	1.05	0.72	0	0.04	0.15	0.33	0.48	1.24	3.92	25
1/1852-12/1856	33	0.76	0.98	0.55	3	-0.43	0.11	0.35	0.75	1.56	1.98	19
1/1857-12/1861	54	0.98	1.15	0.55	2	-0.19	0.28	0.71	0.99	1.55	2.74	27
1/1862-12/1866	72	0.83	1.01	0.73	4	-0.86	0.20	0.40	0.70	1.57	3.70	12
1/1867-12/1871	76	0.74	0.76	0.58	8	-0.91	0.09	0.42	0.67	1.35	2.36	11
1/1872-12/1877	82	1.05	0.96	1.01	10	-0.71	0.01	0.30	0.85	2.59	3.55	15
1/1877-12/1881	113	1.29	0.96	1.50	12	-3.54	-0.03	0.32	0.93	3.14	7.80	17
1/1882-12/1886	154	1.03	1.03	1.44	18	-1.33	-0.35	0.14	0.66	2.90	7.94	6
1/1887-12/1891	161	1.52	0.74	1.73	14	-0.76	-0.05	0.10	0.94	4.03	7.81	22
1/1892-12/1896	196	1.24	0.95	1.32	12	-1.74	-0.02	0.34	1.04	2.87	7.23	8
1/1897-12/1901	252	1.13	0.93	1.11	8	-3.03	0.02	0.27	1.09	2.38	5.47	14
1/1902-12/1906	374	1.24	1.03	1.36	11	-1.33	-0.08	0.33	1.15	2.63	8.92	10
1/1907-12/1911	424	1.16	1.05	1.18	9	-2.14	0.01	0.34	1.06	2.45	7.33	12

Note: This table displays cross-sectional descriptive statistics of betas estimated by market model (1) of returns on their market counterparts over the fifteen five year sub periods between January 1832 and December 1911. The number of stocks in the various periods ranges from 21 to 424 for stocks with full return data within a period. The table also reports the percentage of beta sample in a period that is less than zero. The maximum and minimum beta of each period is also recorded. The last column reports the average coefficient of determination $\left(R^2\right)$. Equally weighted (EQ) and value weighted (VW) mean betas are also displayed in the table. The value weighted mean is the average of the betas weighted by their market capital at the beginning the period.

The figures in the last column show the average coefficient of determination $\left(R^2\right)$ in percentages, which is a measure of explanatory power of the market model. Surprisingly, the literature reports negative betas on different markets after World War I. For example, Altman, Jacquillat and Levasseur (1974), Dimson and Marsh (1983) recorded a large number of negative betas in their weekly returns interval estimation of beta on the French and the American markets.

Table 2. Average Beta and Average Coefficient of Determination of the sized based sub-samples

Period	Large mean(β_{iL})	Large mean R^2(%)	Medium mean(β_{iM})	Medium mean R^2(%)	Small mean(β_{iS})	Small meanR^2(%)
1/1837-12/1841	1.29	30	1.73	33	1.07	23
1/1842-12/1846	1.19	37	0.79	11	1.40	10
1/1847-12/1851	0.94	43	0.49	22	0.67	10
1/1852-12/1856	1.05	35	0.66	14	0.61	10
1/1857-12/1861	1.15	40	1.08	27	0.69	13
1/1862-12/1866	1.04	23	0.76	8	0.70	5
1/1867-12/1871	0.77	17	0.59	9	0.90	7
1/1872-12/1877	1.06	19	1.02	16	1.10	10
1/1877-12/1881	1.03	20	1.51	18	1.30	12
1/1882-12/1886	1.01	9	1.16	6	0.86	3
1/1887-12/1891	0.84	18	1.73	28	1.95	18
1/1892-12/1896	1.00	10	1.29	7	1.40	6
1/1897-12/1901	0.98	18	1.22	15	1.16	9
1/1902-12/1906	1.06	15	1.27	9	1.39	4
1/1907-12/1911	1.18	20	1.17	11	1.13	5

Note: In these table stocks in each period was sorted in descending order based on their market capital. The first 30% of stocks in the periods are classified as large stocks, the next 40% as medium stocks and last 30% as small stocks. Cross-sectional statistics of the betas of the sub-samples were computed and their equally weighted averages of betas and coefficient of determination $\left(R^2\right)$ are displayed. The L, M and S subscripts on betas indicates Large, Medium and Small stocks respectively.

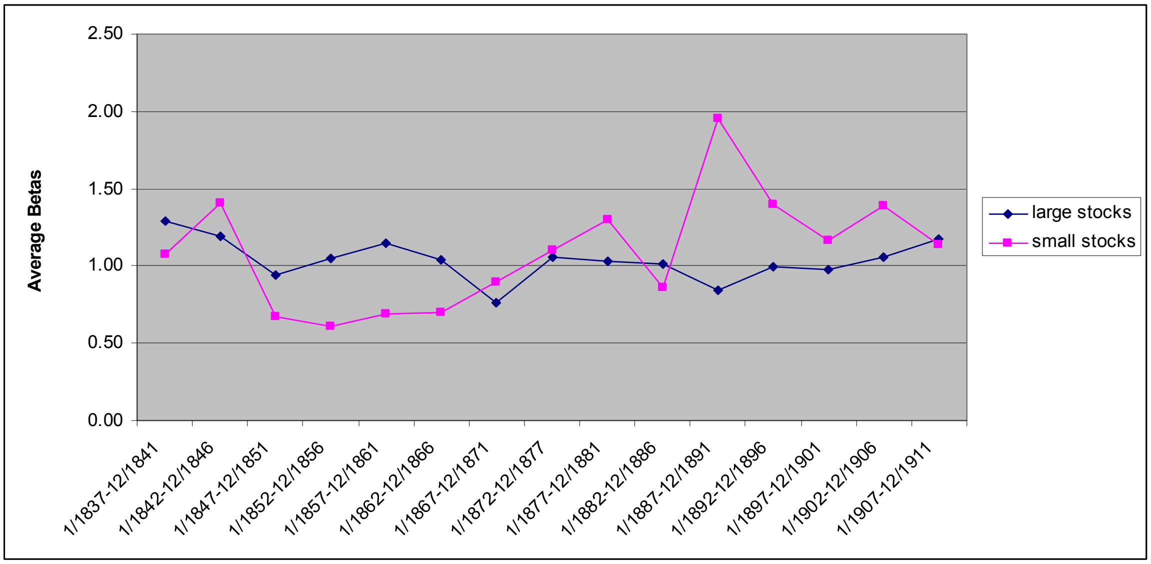

Figure 1. Graph of the average beta of each period for large stocks and small stock

Note: The average beta estimates for large and small stocks are plotted against their corresponding periods of estimation. The diamond points indicate the averages of large stock and the square points for small stocks. This depicts the extreme values of beta estimates recorded for the small stocks relative to the benchmark beta of one.

It is also important to note the very high and low betas in the 19th century BSE compared to the post-1926 market betas recorded in the literature. The possible explanation for these extreme values of the beta and low coefficients of determination might be linked to the infrequent trading effect and the influence of extreme observations (outliers) in the returns series of the stocks.

In order to capture the effect of size (market capitalization) on the average beta and R^2 values, we computed the same cross-sectional statistics for betas in each period. However, the stocks in each period are sub-divided into three mutually exclusive size-based sub-samples. The composition of the sub-samples is as follows: the sample of stocks in each period is sorted in descending order based on their market capitalization (size) at the beginning of the period.

The first 30 percent of stocks are classified as the large stocks portfolio, next 40 percent as medium stocks portfolio, and the last 30 percent as the small stocks portfolio. Table 2 reports the average beta and the R^2 values for each sub-sample in each period. Generally, the high values of R^2 recorded for large stocks are striking because on the UK market, Dimson (1979) recorded a similar pattern with 15 years of monthly data. This might be attributed to the value-weighted index used in the computation, since the index will have more explanatory power for large-sized stocks. Figure 1 depicts the cross-sectional average beta of each period for large stocks and small stocks. It is clear from Figure 1 that small stocks record comparatively low betas in the periods before 1867 and high betas thereafter. The result in the periods before 1867 corroborates the assertion of Dimson (1979), Scholes and Williams (1977) and Beer (1997). They said that if trading frequency is highly correlated with the market capitalization of the stock, betas of small stocks (infrequently traded stocks) are lower when estimated with the market model. On the contrary, Ibbotson et al. (1997), based on NYSE data between 1926 and 1994, found that small stock returns due to infrequent trading show a high degree of autocorrelation and that they are capable of recording high betas. On the 19th century BSE, we confirm their result in the periods after 1867. The only anomaly is the period between 1882 and 1886, where the small stock average beta lies slightly below the average beta of large stocks.

5. Beta Stability

Here, we adopt the Blume (1971) and Altman et al. (1974) correlation method of investigating the stability of beta estimates. Blume (1975) shows that the beta coefficient between two successive periods is stationary if (Note 2) $E\left(\beta_{p_i}\right)=E\left(\beta_{p_{i+1}}\right), Var\left(\beta_{p_i}\right)=Var\left(\beta_{p_{i+1}}\right),\ corr\left(\beta_{p_i},\beta_{p_{i+1}}\right)=1$ where β_{p_i} are the betas in period p_i. Betas in period p_i are used to rank stocks existing in periods p_i and p_{i+1} in ascending order. In period p_i equally weighted portfolios of $s=1,2,3,\ldots$ stocks are formed as follows: the first portfolio consists of stocks with s smallest beta estimates. The next portfolio consists of stocks with the next smallest beta estimates. This process of portfolio formation is repeated until the number of stocks left is less than *s*. For each *s*, the betas of all portfolios in period p_{i+1} are also computed. We compute the correlation and Spearman rank order correlation of the betas between each two adjacent periods. Table 3 reports the weighted average correlation across all the sub-periods studied. The weighted average correlation takes into account the number of stocks or portfolios in each adjacent ten-year period. The weighted average correlation across the successive periods ranges from 0.54 for individual stocks to 0.95 for portfolios of 20 stocks. These values indicate that the beta of individual stocks have, on average, less information about their future beta than the portfolio beta. Blume (1971) found a similar result on the US market. Due to the limited number of stocks, we were not able form up to 50-sized portfolios on the BSE. Blume finds 0.62(0.67) and 0.91(0.93) mean correlation (rank correlation) for 1- and 10-sized portfolios using 84-month estimation periods respectively. Furthermore, based on 52-week estimation periods, Levy (1971) records 0.44 and 0.82 mean correlation for 1- and 10-sized portfolios on the same exchange. On the UK market between 1955 and 1979, Dimson and Marsh (1983) used value-weighted market index, monthly returns interval measurement and sixty-month estimation periods to obtain an average correlation of 0.56 and 0.91 for 1- or 10-sized portfolios respectively. The correlations in Table 3 show that the betas from the 19th century BSE are stable compared to the betas in post-1926 US and UK markets.

Table 3. Weighted Average of Correlation and Spearman Rank Order Correlation across successive Periods

No. of Stock per Portfolio	Correlation	Speaman's rank Correlation
1	0.54	0.56
2	0.62	0.64
4	0.72	0.72
7	0.76	0.77
10	0.92	0.91
20	0.95	0.96

Note: This table shows the weighted average correlation and Spearman's Rank correlation of betas of individual stock and portfolios in successive periods across the fifteen periods studied. For a stock to be included in this analysis it has data for completely two consecutive periods. Betas in a period (estimation period) are used to rank the betas in that period and the next adjacent period (prediction period) in ascending order. Portfolios are formed with their constituent as follows: The first portfolio is the first s stocks for $s = (1,2,4,7,10,20)$. The second portfolio contains the following s stocks and so on until available stocks is less than s. Assuming equal amount is invested in each stock then the portfolio beta will just be the mean of the betas of stocks included in the portfolio. We computed the weighted average correlations weighted by the number of portfolios in each ten year period.

6. Blume and Vasicek stability adjustment techniques

Individual stock betas estimated from the MM are noted as unstable in the previous section (also in Blume (1971), Collins, Ledolter and Rayburn (1987), Faff and John (1992), Gregory-Allen et al. (1994), Eisenbeiss, Kauermann and Semmler (2007)). There is a tendency for a high beta estimate to overstate its true value and vice versa. Therefore, we use the Blume (1971) autoregressive adjustment model to improve the stability of beta estimates for both individual stocks and portfolios.

Table 4. Measurement of Regression Tendency of Estimated Beta Cofficients for Individual Stocks

Regression Tendency Implied Between Periods			$\beta_{jp}=a+b\beta_{jp-1}+\eta_p$				
			a	b	R^2 (%)	H_0: b=0	H_0: b=1
1/1842- 12/1846	and	1/1837- 12/1841	1.40	0.00	0	0.00	-1.31
1/1847- 12/1851	and	1/1842- 12/1846	0.74	-0.07	3	-0.96	-14.11
1/1852- 12/1856	and	1/1847- 12/1851	0.48	0.33	20	2.58	-7.60
1/1857- 12/1861	and	1/1852- 12/1856	0.53	0.54	26	3.22	-5.41
1/1862- 12/1866	and	1/1857- 12/1861	0.27	0.55	18	3.32	-5.43
1/1867- 12/1871	and	1/1862- 12/1866	0.63	0.13	3	1.38	-10.34
1/1872- 12/1876	and	1/1867- 12/1871	0.77	0.43	5	1.94	-4.04
1/1877- 12/1881	and	1/1872- 12/1876	0.65	0.70	20	3.99	-5.03
1/1882- 12/1886	and	1/1877- 12/1881	0.44	0.67	39	7.80	-10.91
1/1887- 12/1891	and	1/1882- 12/1886	0.86	0.69	31	7.66	-10.49
1/1892- 12/1896	and	1/1887- 12/1891	0.58	0.39	27	7.42	-18.75
1/1897- 12/1901	and	1/1892- 12/1896	0.60	0.38	27	7.81	-19.99
1/1902- 12/1906	and	1/1897- 12/1902	0.49	0.70	28	8.94	-11.98
1/1907- 12/1912	and	1/1902- 12/1906	0.78	0.39	15	7.67	-19.21
All periods			**0.67**	**0.46**	**20**	**18.75**	**-41.46**

Note: In this table beta of stock existing in a period are regressed on the betas of the same stocks in a prior adjacent period. R^2(%) is the percentage of the variance of betas in the period p explained by betas in the period p-1. The t-statistic of th null hypothesis of the slope coefficient (b) equal to zero or one is also reported in the last two columns.

Table 4 presents regression tendencies implied between adjacent periods, where a and b are the constant term and slope coefficients, respectively. The values of the coefficients in the periods between 1837 and 1867 are striking. It is not consistent with the Blume assertion that all the coefficients lie between zero and one. The last two Columns show the t-statistics of the test of a hypothesis of the slope coefficient equal to zero or one. The t-statistics of the slope coefficients show that the null hypothesis of the slope coefficient equal to zero is rejected in the adjacent periods after 1872. In addition, the null hypothesis of the slope coefficient equal to one is rejected in all the adjacent periods (except the first two periods). The R^2 values also show that the betas in the periods after 1872 have more explanations for their prior betas than those before 1872. As can be seen in Table 4, the coefficients change over time, but there are extreme coefficients outside the interval between zero and one in the first two periods. The extreme values may be attributed to the number of stocks in a period as we record less than 50 stocks in our first two five-year periods. A result not reported shows that increasing the length of the estimation period (such as seven years in Blume (1971)) improves the R^2 values and the t-statistics but at the expense of losing more stocks, since fewer stocks have complete returns' data for longer periods.

With the regression tendencies, suppose we want to forecast the beta for any stock or portfolio in the period 1842-1846. We compute its beta in 1837-1841. The forecast of the beta is obtained by substituting it for $\beta_{p\text{-}1}$ in equation (2) with the coefficients in the first row of Table.4. β_p is then computed from the equation and used as the forecast. The adjustment process is repeated for stocks in the subsequent 13 adjacent periods using their respective coefficients. We also introduce the Vasicek adjustment model (3) to adjust betas in successive adjacent periods. We test the predictive performance of the various adjusted betas by using the root mean square error (Note 3) (RMSE) criterion. The RMSE tests the performance of the autoregressive methods based on variation and unbiasedness of their beta forecast. The adjustment method is repeated for all adjacent periods on equally weighted portfolios of size 2, 4, 7, 10 and 20. In order to compare the predictive performance of the MM betas and the autoregressive-adjusted betas, we compute the RMSE of betas estimated with MM in adjacent periods. Table 5 displays the average RMSE of the adjusted betas and MM betas across the 15 periods studied. It is clear from the Table that the average RMSE of the adjusted betas is lower than that of the market model betas. Table 5 also shows that the predictive performance improves as the number of stocks in a portfolio increases for both adjusted and the MM betas. For individual stocks, the Bayesian adjustment technique proposed by Vasicek is superior to Blume's adjustment as reflected in the small average RMSE. Blume (1971) and Klemkosky and Martin (1975) recorded similar patterns of the predictive performance on the NYSE market. Their adjusted betas mean square errors were smaller than their MM betas. For portfolios of size 7 or more, one cannot see much difference between the Blume adjustment method and the Bayesian approach.

Table 5. Predictive Performance of Blume and Vasicek (Bayesian) procedures of estimating Beta

No.of Stock		Average RMSE	
per Portfolio	MM	Blume	Bayesian
1	1.08	0.57	0.48
2	0.76	0.44	0.38
4	0.57	0.35	0.31
7	0.45	0.29	0.27
10	0.40	0.23	0.27
20	0.25	0.19	0.23

Note: Average RMSE and MAE across the fifteen periods studied were used to compare the predictive performance of the various adjusted betas and the market model (MM) betas in successive periods. The average RMSE and MAE across the successive adjacent periods for the various equally weighted portfolio formations are displayed.

The conclusion is that on the 19th century BSE, the predictive accuracy of betas estimated by the MM can be improved by adjusting betas using either the Blume or Bayesian adjustment methods and a portfolio with a sizeable number of stocks.

The reliability of the conclusion above can be confirmed by performing an additional test on the root mean squared error values. The possible method is to test whether the differences in the values of the RMSE's are statistically significant. Harvey et al. (1997) presented a modified Diebold and Mariano test statistic that will be used for this purpose. Therefore, suppose we want to compare the forecasts of Blume (*BL*) and Vasicek (*VA*) models. $\left\{\varepsilon^{BL}\right\}$ and $\left\{\varepsilon^{VA}\right\}$ are the forecasting errors from the Blume and Vasicek models, respectively. In our case, we consider the root mean square error function, $f\left(\varepsilon^{BL}\right)=$ root mean square error of the Blume adjusted betas. The test is based on the loss differential function $d_j = f\left(\varepsilon_j^{BL}\right) - f\left(\varepsilon_j^{VA}\right)$ for $j = 1, \ldots, H$. The null hypothesis of expected equal predictive performance is H_0: $E\left(d_j\right)=0$ and the alternative hypothesis of the Blume model predicting worse than the Vasicek model is H_a: $E\left(d_j\right)>0$. The Modified Diebold-Mariano (MDM) statistic is:

$$MDM=\left[\frac{H+1-2h+H^{-1}h\left(h-1\right)}{H}\right]^{1/2}.\frac{\overline{d}}{\left(\lambda_0+2\sum_{i=1}^{\infty}\lambda_i\right)^{1/2}}\sim t_{v,H-1}\left(0,1\right),$$

where $\overline{d}=H^{-1}\sum_{j=1}^{H}d_j$, $\lambda_i=\text{cov}\left(d_j,d_{j-i}\right)$, h is the horizon of forecast and $t_{v,H-1}(0,1)$ a student's t distribution with $H-1$ degrees of freedom and v is the significant level usually set at 5%. The test compares the Diebold-Mariano test statistics to critical values from the student's t distribution. We reject the null hypothesis of equal predictive accuracy when the test statistic is greater than the critical value at v level. In order to apply this test, betas estimated with the Market, Blume and Vasicek models in all periods are pooled together to form three series of length *H*. Then we perform the test on the three series.Table 6 reports the modified Diebold-Mariano test statistics between the various models under study. Betas estimated from the various models are considered across the entire period studied.

Table 6. Modified Diebold-Mariano test statistics (p-value in parentheses). H_0:$E(d_j)=0$ and H_a:$E(d_j)>0$

Models	Strict Regulatory period 1/1837-12/1871	Deregulation and expansion period 1/1872-12/1911	Overall period 1/1837-12/1911
β_{MM} vs. β_{BL}	**2.87**	**1.99**	**2.18**
	(0.00)	(0.02)	(0.01)
β_{MM} vs. β_{VA}	**1.44**	**4.11**	**3.75**
	(0.08)	(0.00)	(0.00)
β_{BL} vs. β_{VA}	**0.19**	**0.78**	**0.56**
	(0.42)	(0.22)	(0.29)

Note: These table reports the modified Diebold-Mariano test statistics for one step ahead equal forecast accuracy between the Market Model, Blume and the Vasicek's adjusted betas. β_{MM}=market model betas, β_{BL}=Blume's adjusted betas, β_{VA}=Vasicek's adjusted betas.

Once again the period studied is divided into two sub-periods based on the environment in which the BSE operated, a period of strict regulation and a period of deregulation and expansion. The values in the first row of the Table reveal that we can confidently reject the null hypothesis of one-step ahead equal predictive accuracy of the Blume and MM betas. For instance, in the overall period of our sample the null hypothesis can be rejected at the 5 percent level. During the strictly regulated period, we find that Blume-adjusted beta significantly outperforms the market model beta. Between the Vasicek adjusted betas and the MM betas, we can reject the null hypothesis of equal predictive accuracy for the deregulated and expansion period. The significance level of the rejection becomes weak in the strictly regulated period. The equality in the predictive performance of the Vasicek and the MM betas is strongly rejected in the entire period. The values from the bottom row of Table 6 show that there is no significant difference between the Vasicek betas and the Blume betas in terms of their one-step ahead forecast. Therefore, we cannot reject the null hypothesis of equal predictive accuracy between the two models.

7. Beta Bias

Considering the period of the study and the trading frequency of the market, we might expect that some stocks may not trade every month for economic reasons or because of regulatory conditions. These stocks may systematically lead or lag the market movement, producing biased betas when beta is estimated with the MM. In order to expose the presence of possible lead or lag effects, we test the significance of the coefficient of the returns on the lagged or lead market index. The Dimson bias adjustment equation with maximum lag or lead of three months is considered, that is $i=-3,...,3$. We use only three months lag and lead because beta bias has been documented as not prevalent in monthly returns data (see Cohen, Hawawini, Maier, Schwartz and Whitcomb (1983)). For each stock, the estimates of the parameters β_i indicate the lagged, matched and lead beta coefficients. We test the hypothesis $H_0:\beta_i=0$ against the alternative $H_1:\beta_i\neq 0$ for each stock. Table 7 reports the cross-sectional average of the lag and lead betas in each period. The numbers in parentheses are the percentage of stocks that reject the null hypothesis.

Evidence from this Table indicates that beta coefficients β_i for $i \neq 0$ are not significantly different from zero for the majority of the stocks. This shows that the explanatory power of the model for $i \neq 0$ is approximately zero for most of the stocks. Unsurprisingly, there are some stocks with lead and lag coefficients that are statistically significant, but their numbers does not exceed the coefficients corresponding to the match. This indicates that there is no severe timing problem in the 19th century data.

As most of the lead and lagged coefficients are significantly equal to zero, we can interpret this as evidence of the market model (MM) producing statistically reliable beta estimates in relation to the other models, which incorporate the lagged and lead market indexes. These results can be compared to the results from the post-World War I markets. For example, Hawawini and Michel (1979) found a similar pattern of results on the Belgium stock exchange by using weekly interval returns data between 1963 and 1976. The result also follows Cohen et al. (1983) hypothesis that there is a strong relationship between beta estimates and the length of the interval over which returns are measured. They established that beta bias mostly shows up in the short length interval (daily) of returns, and the bias disappears when the difference of the interval is lengthened (monthly). Similarly, on the New Zealand market, Bartholdy and Riding (1994) used monthly data to establish that betas estimated from MM are less biased. On the contrary, Ibbotson et al. (1997) reports that lagged coefficients should be considered when estimating beta.

8. Impact of Outlying observations on Beta

The extreme (maximum/minimum) beta estimates recorded by some stocks in Table 1 for the five-year periods studied might be due to the influence of outliers or unexpected movement by the stock or the market returns. The literature shows that outliers have a tendency to reduce or increase the magnitude of the beta when it is estimated with the MM (Chatterjee and Jacques (1994)). In such cases, reducing the impact of outliers in the estimation of the beta can significantly change the value of beta. We apply the IRLS (outlier resistant), which minimizes a weighted sum of squares of residuals. The weights given to each return pair observation depends on the distance between the observation and the fitted line (Martin and Simin (2003)). Table 8 reports how the presence of outliers affects the beta value. The number of stocks in each five-year period is grouped into two. As explained in previous section, stocks with identified outliers less than or equal to 4 are grouped into Category A and those with identified outliers greater than 4 are grouped in category B. We compute the average beta of each category.

Table 7. Dimson Aggregate Coefficient (AC) beta Adjustment

Period	Number of stocks	AC Beta	Mean Lag,Match and the Lead beta estimates						
			β_{-3}	β_{-2}	β_{-1}	β_0	β_1	β_2	β_3
1/1837-12/1841	21	1.37	0.03	0.05	-0.12	1.42	-0.07	-0.02	0.07
			(10)	(5)	(5)	(95)	(0)	(0)	(5)
1/1842-12/1846	33	1.47	-0.11	0.37	0.10	1.06	0.12	-0.10	0.03
			(3)	(12)	(18)	(58)	(15)	(0)	(6)
1/1847-12/1851	32	0.69	0.00	0.07	-0.16	0.76	-0.03	0.07	-0.02
			(6)	(13)	(3)	(72)	(13)	(13)	(3)
1/1852-12/1856	33	0.80	0.06	0.03	0.00	0.76	0.01	-0.07	0.01
			(3)	(6)	(0)	(73)	(6)	(6)	(0)
1/1857-12/1861	54	0.97	0.00	-0.02	0.07	0.99	-0.10	0.00	0.04
			(2)	(7)	(6)	(78)	(4)	(2)	(11)
1/1862-12/1866	72	0.56	-0.21	0.19	-0.09	0.90	-0.16	0.05	-0.12
			(4)	(10)	(8)	(42)	(0)	(3)	(0)
1/1867-12/1871	76	0.49	-0.09	-0.15	0.05	0.73	0.04	0.00	-0.08
			(1)	(3)	(5)	(67)	(3)	(3)	(3)
1/1872-12/1877	82	1.17	0.07	-0.08	0.24	1.02	-0.17	0.12	-0.02
			(6)	(7)	(13)	(55)	(6)	(11)	(11)
1/1877-12/1881	112	1.67	0.12	-0.05	0.10	1.19	0.07	0.09	0.15
			(6)	(7)	(13)	(55)	(6)	(11)	(11)
1/1882-12/1886	154	0.74	-0.03	-0.03	-0.01	0.97	0.11	-0.10	-0.18
			(6)	(3)	(6)	(38)	(9)	(5)	(1)
1/1887-12/1891	160	1.68	0.03	0.12	-0.09	1.58	-0.04	0.00	0.08
			(4)	(7)	(5)	(52)	(7)	(4)	(6)
1/1892-12/1896	196	1.42	0.07	-0.07	0.12	1.19	0.08	0.08	-0.04
			(6)	(6)	(6)	(42)	(8)	(5)	(4)
1/1897-12/1901	252	0.96	-0.07	-0.05	0.05	1.17	-0.18	0.06	-0.02
			(9)	(5)	(6)	(59)	(4)	(8)	(7)
1/1902-12/1906	374	1.11	-0.04	0.09	0.22	1.18	-0.03	-0.16	-0.15
			(4)	(7)	(10)	(47)	(3)	(3)	(4)
1/1907-12/1911	424	1.23	-0.03	0.04	0.12	1.13	0.04	-0.05	-0.01
			(2)	(3)	(10)	(56)	(5)	(6)	(5)
Overall periods	2075	1.09	-0.01	0.03	0.04	1.07	-0.02	0.00	-0.02
			(5)	(6)	(8)	(53)	(5)	(5)	(5)

Note: This table reports Dimson's aggregate coefficient adjusted beta in each five year period. The numbers in parenthesis are the percentage of stocks in a period that reject the null hypothesis of the coefficient been zero at 5% significant level.

In each period, we compare the cross-sectional average betas of the MM and IRLS for each category. For example, in the first period out of the 21 stocks, three fall in Category A with an average market model (MM) beta estimate of 1.29 and IRLS beta of 0.46. In Category A, the difference between the average MM beta and the average IRLS beta is 0.83. Looking across periods, except for periods 1 to 3, the rest of the periods have more stocks in Category A than Category B. In each period, the difference between the MM betas and the IRLS betas for Category B is greater than the difference in Category A (last column). This implies that the more the outlier observations in the return series, the higher the market model overestimates beta. The MM beta is always greater than the IRLS beta in Category A (across periods in Figure 2). It confirms the result by Chatterjee and Jacques (1994) that the weighted least squares estimation reduces the MM betas by a certain percentage. We apply the modified Diebold-Mariano test to compare one-step ahead predictive accuracy of the MM and IRLS estimated betas.

Table 8.

Period	Total Number of stocks	Number of detected outlier observations	Number of Stocks	Average MM	Average IRLS	β_{mm}-β_{IRLS}
1/1837-12/1841	21	**A**	**3**	**1.29**	**0.46**	**0.83**
		B	(18)	1.38	0.19	1.19
1/1842-12/1846	33	**A**	**10**	**0.72**	**0.49**	**0.23**
		B	(23)	1.26	0.03	1.24
1/1847-12/1851	32	**A**	**9**	**0.60**	**0.52**	**0.08**
		B	(23)	0.72	0.26	0.46
1/1852-12/1856	33	**A**	**23**	**0.97**	**0.81**	**0.16**
		B	(10)	0.28	0.07	0.21
1/1857-12/1861	54	**A**	**41**	**1.10**	**1.00**	**0.10**
		B	(13)	0.60	0.10	0.51
1/1862-12/1866	72	**A**	**48**	**0.88**	**0.60**	**0.28**
		B	24	0.72	0.15	0.57
1/1867-12/1871	76	**A**	**42**	**0.79**	**0.56**	**0.23**
		B	(34)	0.67	0.06	0.61
1/1872-12/1877	82	**A**	**54**	**1.21**	**0.80**	**0.42**
		B	(28)	0.75	0.14	0.61
1/1877-12/1881	112	**A**	**78**	**1.47**	**1.05**	**0.42**
		B	(34)	0.91	0.20	0.71
1/1882-12/1886	154	**A**	**110**	**1.19**	**1.13**	**0.06**
		B	(44)	0.62	0.05	0.57
1/1887-12/1891	160	**A**	**120**	**1.77**	**1.41**	**0.36**
		B	(40)	0.80	0.16	0.63
1/1892-12/1896	196	**A**	**146**	**1.45**	**1.06**	**0.39**
		B	(50)	0.62	0.12	0.50
1/1897-12/1901	252	**A**	**198**	**1.33**	**1.07**	**0.26**
		B	(54)	0.40	0.07	0.34
1/1902-12/1906	374	**A**	**290**	**1.36**	**1.07**	**0.29**
		B	(84)	0.83	0.19	0.64
1/1907-12/1911	424	**A**	**346**	**1.32**	**1.12**	**0.20**
		B	(78)	0.47	0.07	0.40

Note: This table reports the number of stocks that has outlier observation points less or equal to 4 (A) and greater than 4(B) (parenthesis) in each period. Its also display the cross-sectional average market model(MM) beta and the iterative reweighted least square(IRLS) betas for this stocks. Column two lists the total number of stocks which has complete data within the period. β_{mm}=cross-sectional average of the market model betas, β_{IRLS}=cross-sectional average outlier of the IRLS betas.

The modified Diebold Mariano test statistics proposed by Harvey et al. (1997) are employed to test the null hypothesis of equal predictive accuracy against the alternative of IRLS betas forecasting better than the MM betas. A pooled sample of betas within the period studied is considered. The modified Diebold- Mariano's test statistic between the two models is 1.37 with p-value of 0.09 for one-step forecasts in the period of our study (shown in Table 9). This shows that we cannot reject the null hypothesis of equal predictive accuracy at the 5 percent significance level in the overall period. The null hypothesis can be rejected only at the 10 percent level. From the deregulation and expansion period, the null hypothesis of equal predictive accuracy is not rejected. From Table 9, we conclude that on the 19th century BSE, the IRLS method can help to curb the influence of outliers on estimated betas, but it does not significantly outperform the standard MM in terms of their ability to predict one-step ahead in the period of deregulation and expansion.

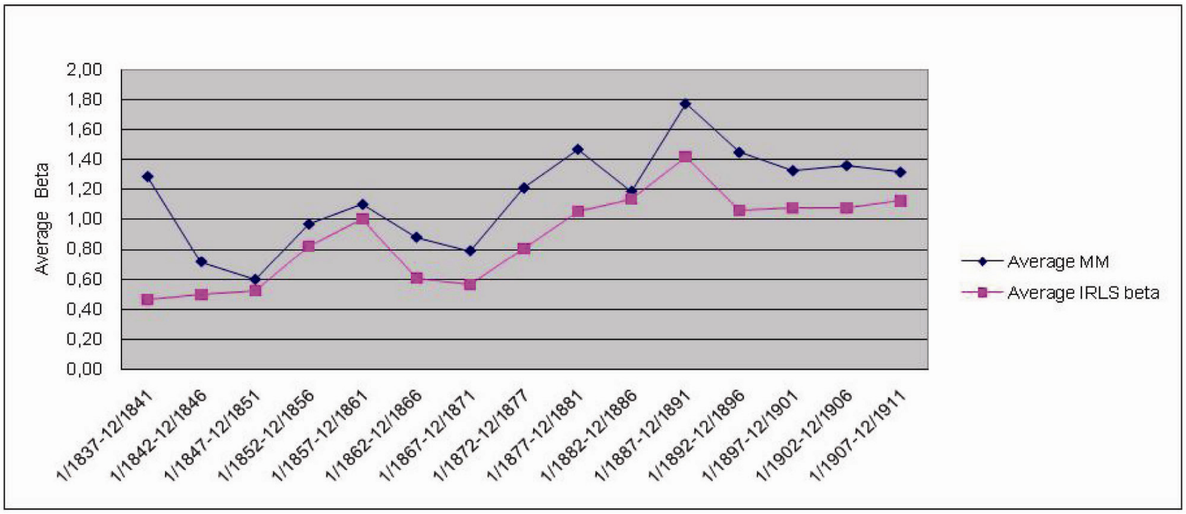

Figure 2. Plot of average MM/IRLS betas for stocks with outlier observations less than 4

9. Conclusions

This study evaluates the relative performance of different methods of estimating beta based on their ability to predict subsequent beta on the 19th century BSE. The analysis of the different beta techniques reveals that beta estimated with the market model is not stable. Specifically, the study reveals that for individual stocks, the market model beta is weak in its ability to predict the future beta. The predictability can be improved by grouping 10 or more stocks to form a portfolio or adjusting betas with the Vasicek and Blume autoregressive techniques. The study also shows no significant difference between the Blume and Vasicek adjusted betas in terms of their predictive accuracy. Applying the Dimson method, correcting nonsynchronous trading effect reveals that returns of few stocks have a significant relationship with the lead and lag market returns. There is no significant difference in the predictive accuracy of the betas estimated with the IRLS method and the market model in the deregulation and expansion period. In the next chapter, we study the ability of beta to explain returns in the cross-section of stocks, which is the primary implication of the CAPM.

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Notes

Note 1. $E(x)$ is the expected value of x , $Var(x)$ is the variance of x

Note 2. $corr(x, y)$ is the correlation between x and y .

Note 3. The root mean square error was calculated by $\sqrt{\sum\left(\beta_{jt_i} - \beta_{jt_{i-1}}\right)^2 / N}$ for $j = 1,..., N$