The Anomalous Behavior of Stock Prices on the Canadian Securities Exchange

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Received: August 13, 2024 Accepted: September 25, 2024 Online Published: October 6, 2024

doi:10.5430/ijfr.v15n4p1 URL: https://doi.org/10.5430/ijfr.v15n4p1

Abstract

This paper examines the conformity of the distribution of stock prices on the Canadian Securities Exchange (CSE) to that theorized by Benford's Law (BL). BL follows a logarithm law such that the leading digits have a higher probability of lower numbers such as 1, 2, or 3 versus higher numbers like 7, 8, or 9. This analysis can be used to detect fraudulent manipulation of stock prices. Previous research has applied BL to a broad range of data such as cost data, atomic weights, river areas, and populations, as well as stock prices. After collecting stock prices on the CSE, the number count for each digit was compared to the expected number given the frequencies posited by BL. A chi-square test was employed to determine statistical significance. The first digit was found to adhere to BL; however, the second digit was not congruent with that predicted by BL. There is an indication of possible manipulation on this stock exchange. These mixed results are consistent with the empirical evidence of other researchers. This evidence is relevant to auditors, shareholders, financial analysts, investment managers, government, and the CSE.

Keywords: Benford's Law, fraud detection, Chi-square, stock prices

1. Introduction

There is a growing body of literature observing the distribution of numbers (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) in data by the digit place (first digit, second digit, third digit and fourth digit). That is, the data point 36.47 has 4 digits. The 1st digit is the number 3, the 2nd digit is the number 6, the 3rd digit is the number 4, and the 4th digit is the number 7). While a data point may have more than 4 digits, the analysis typically ends at the fourth digit. If the data point indicates a zero as the first digit, that is ignored and is not treated as the first digit. For example, 0.218 has the 1st digit as the number 2, the 2nd digit as the number 1, and the 3rd digit as the number 8.

One may expect that each number is equally likely to occur for each digit. Newcomb (1881) posited that the probability of each number by digit was not equal but instead follows a logarithmic law. Benford (1938) set out to examine this suggested logarithmic law by collecting 20,229 data points from 20 disparate data sets, such as cost data, atomic weights, river areas, populations, and river drainage rates. Since this publication, the digit frequency phenomenon has been known as Benford's Law (BL).

This research studies the prices of stocks listed on the Canadian Securities Exchange (CSE). The CSE is of particular interest as the stock prices are low and little research has been conducted on this stock exchange. The actual frequencies of the numbers by digit place will be compared for compliance to that predicted by BL.

2. Literature Review

The BL field of study is devoid of an explanation as to why the digit frequencies by number follow a logarithm function. Nonetheless, authors have approached this question in other ways. Hill (1995a, 1995b, 1998) ruminated over numerous mathematical derivations underpinning BL with no conclusive theory. Pietronero et al. (2001) explain the recurrence of California earthquakes with BL (where alpha equals 1) and how it relates to multiplicative processes. An extrapolation to Zipf's Law (where alpha equals 2) is also developed. Becker et al. (2018) investigate the application of BL to the decomposition of conserved quantities and hypothesize about fragmentation processes.

Durtschi et al. (2004) demonstrated how auditors can scrutinize disbursements and receipts considering BL frequencies as the norm. Saville (2006) used BL to identify false reporting of accounting data on the Johannesburg Stock Exchange. Kuruppu (2019) articulated a methodology of how auditors can detect economic crime in companies. Ozevin and Yazdifar (2020) inspect financial statement data to assess for conditions leading to fraud risk and employing BL. Todter (2009) applied BL to published articles in the economics literature, checking for academic fraud. Surprisingly, approximately 30 percent of articles were classified as suspected of fraud. Here, fraud was defined where "scientists commit fabrication, falsification, or plagiarism". Note, these results were for suspicion of fraud; not conclusive evidence.

A selected body of knowledge surveying the conformity of stock prices to BL has De Ceuster, Dhaene, and Schatteman (1998) employing BL on the trailing digits of the Dow Jones 30 Industrial Average, Financial Times Stock Exchange 100, and the Nikkei Stock Average 225 indexes. The distributions were congruent with BL and supported the finding of no psychological barriers. Riccioni and Cerqueti (2018) applied BL to stock prices and share volume on one day on stock exchanges in the US, Belgium, Denmark, France, Germany, Italy, Norway, Sweden, Switzerland, Indonesia, New Zealand, Singapore, Hong Kong, Argentina, and Brazil with mixed results matching the predicted probabilities. Jayasree, Jyothi, and Ramya (2018) found little support for India Nifty Fifty stocks conforming to BL with respect to stock returns and shares traded.

3. Theory

3.1 Benford's Law

Benford postulated that the expected occurrence of a number as the first significant digit d is:

$$P(d\ is\ the\ first\ digit) = log_{10}\left(\frac{1}{d}+1\right), d = 1,2,\ldots,9 \qquad \text{Equation 1}$$

The general form of Benford's law that specifies the probabilities of occurrence of the first and higher significant digits, and more generally, the joint distribution of all the significant digits is:

$$Prob(n,d) = \left(\sum_{j=10^{n-2}}^{10^{n-1}-1} log_{10}\left(1+\frac{1}{10j+d}\right)\right),$$

where n is the nth leading digit $n > 1$, d is an integer in $\{0,1,\ldots,9\}$. Equation 2

Table 1 contains the Benford's Law distribution for the first four digits, i.e. the expected occurrences of each number zero, one, two, three, four, five, six, seven, eight, and nine in the first, second, third, and fourth (digit) places.

3.2 Hypotheses

We state the null hypothesis as follows:

H_{0i}: The occurrence of numbers in the ith place (digit) of a stock price will conform to the expected Benford's Law distribution,

where i can take the value of 1, 2, 3, or 4. Equation 3

We state the alternative hypothesis as follows:

$H1i$: The occurrence of numbers in the ith place (digit) of a stock price will not form to the expected Benford's Law distribution,

where i can take the value of 1, 2, 3, or 4. Equation 4

The alternative hypothesis posits that the occurrence of numbers in the 1st, 2nd, 3rd, or 4th place (digit) of stock price does not conform to the expected distribution.

4. Methodology

4.1 Sample

The sample is comprised of all the stocks trading on the Canadian Securities Exchange (web address: thecse.com/en/listings) on January 31, April 29, July 29, and October 31 in 2022. The total number of stocks, with at least 1 digit, for each of the 4 dates in chronological order were 615, 643, 667, and 690 respectively. The prices in Canadian dollars (CAD) for each listed stock are collected for a grand total of 2,615 price-dates.

The descriptive statistics are displayed in Table 2.

Table 1. Expected Percentage - Benford's Law

Number	Digit Position			
	1st	2nd	3rd	4th
0	N/A	11.968	10.178	10.018
1	30.103	11.389	10.138	10.014
2	17.609	10.882	10.097	10.010
3	12.494	10.433	10.057	10.006
4	9.691	10.031	10.018	10.002
5	7.918	9.668	9.979	9.998
6	6.695	9.337	9.940	9.994
7	5.799	9.035	9.902	9.990
8	5.115	8.757	9.864	9.986
9	4.576	8.500	9.827	9.982
Total	100.000	100.000	100.000	100.000

Table 2. Descriptive Statistics 2022

	January	April	July	October
n-size	620.000	650.000	678.000	709.000
Minimum	0.005	0.005	0.005	0.005
Maximum	25.100	18.510	16.400	16.170
Mean	0.643	0.486	0.363	0.348
Standard Deviation	2.093	1.468	1.105	1.209
Skewness	8.744	8.559	8.765	8.923
Kurtosis	87.787	86.841	96.458	94.856

4.2 Chi-square Test for Goodness of Fit

Chi-square were collected to determine conformity between the CSE and BL for each of the first 4 digits of the stock price. The null hypothesis is that the digits match BL. The chi-square statistic is calculated as is shown in Equation 5:

$$\chi^2 = \sum_{i=1}^{K} \frac{(AC-EC)^2}{EC}, \quad \text{Equation 5}$$

where AC and EC represent the actual count and expected count respectively, and K represents the number of categories (in our case this equals to 9 for the first digit (as there is no zero number) and 10 for the second, third, or fourth digit respectively). The number of degrees of freedom is $(K-1)$, is 8 for the first digit, and 9 for the second, third, and fourth digits.

The chi-square goodness of fit test has two requirements. First, the observations must be independent. Second, the theoretical expected count must be greater than 5 in at least 80% of the categories, and no category has an expected count less than 1. The chi-square test employed an alpha level of 5%, one tail. For degrees of freedom 8 and 9 the statistical significance criterion values are 15.5073 and 17.5346 respectively.

5. Results

Observing Table 3, Panel A (for January 31, 2022), there are no stocks priced at CAD 100.00 or more. That and there are 147 stocks trading in single digits. Further, there are only 6 stocks trading at CAD 10.00 or more. The CSE is truly a penny stock exchange. Testing the hypothesis that the CSE distribution adheres to the frequencies of BL in Table 3, Panel B, the chi-square first digit statistic of 6.919 is statistically significant at an alpha level of 5%, in support of accepting the null hypothesis. However, the chi-square test statistic for the second digit of 44.181 makes for rejection of the null hypothesis. That is, the alternative hypothesis that the CSE distribution does not conform to that predicted by BL is accepted. The third digit distribution chi-square test statistic of 96.299 is insignificant thereby rejecting the acceptance of conformity to BL. Lastly, the fourth digit distribution does not have a chi-square test statistic reported as it does not meet the requirement of the chi-square test with respect to the theoretical counts.

Table 3. For the Date January 31, 2022

Panel A: Number of Stocks with the Number by Digit Place

Number	Digit Position					
	1st digit	2nd digit	3rd digit	4th digit	5th digit	Total
0	0	80	31	5	0	116
1	183	46	6	0	0	235
2	113	51	4	0	0	168
3	89	33	2	0	0	124
4	64	55	7	0	0	126
5	47	72	3	1	0	123
6	35	25	3	0	0	63
7	38	37	4	0	0	79
8	26	33	3	0	0	62
9	20	36	5	0	0	61
Total	615	468	68	6	0	1,157

Panel B: Percentage of Number and Chi-Square Test Statistic by Digit Place

Number	Digit Position			
	1st digit	2nd digit	3rd digit	4th digit
0	N/A	17.094	45.588	83.333
1	29.751	9.829	8.824	0.000
2	18.374	10.897	5.882	0.000
3	14.472	7.051	2.941	0.000
4	10.407	11.752	10.294	0.000
5	7.642	15.385	4.412	16.667
6	5.691	5.342	4.412	0.000
7	6.179	7.906	5.882	0.000
8	4.228	7.051	4.412	0.000
9	3.252	7.692	7.353	0.000
Total	100.000	100.000	100.000	100.000
Chi-Square	6.919	44.181	96.299	N/A

N/A is not applicable

Looking at Table 4, Panel A (for April 29, 2022) repeats with only 5 stocks priced CAD 10.00 or more. For Panel B, the analysis of the third and fourth digits is set aside as not appropriate due to the chi-square test requirements being violated. Support for BL comes from the first digit with a chi-square statistic of 4.962. Nonetheless, the second digit chi-square statistic of 57.296 fails to comply with the BL hypothesis.

Table 4. For the Date April 29, 2022

Panel A: Number of Stocks with the Number by Digit Place

Number	Digit Position					
	1st digit	2nd digit	3rd digit	4th digit	5th digit	Total
0	0	100	20	2	0	122
1	200	49	3	2	0	254
2	109	51	2	0	0	162
3	83	34	2	0	0	119
4	57	41	3	0	0	101
5	62	57	11	1	0	131
6	36	32	1	0	0	69
7	36	39	3	0	0	78
8	34	27	2	0	0	63
9	26	26	3	0	0	55
Total	643	456	50	5	0	1,154

Panel B: Percentage of Number and Chi-Square Test Statistic by Digit Place

Number	Digit Position			
	1st digit	2nd digit	3rd digit	4th digit
0	N/A	21.930	40.000	40.000
1	31.104	10.746	6.000	40.000
2	16.952	11.184	4.000	0.000
3	12.908	7.456	4.000	0.000
4	8.865	8.991	6.000	0.000
5	9.642	12.500	22.000	20.000
6	5.599	7.018	2.000	0.000
7	5.599	8.553	6.000	0.000
8	5.288	5.921	4.000	0.000
9	4.044	5.702	6.000	0.000
0	N/A	21.930	40.000	40.000
Total	100.000	100.000	100.000	100.000
Chi-Square	4.962	57.296	N/A	N/A

N/A is not applicable

Viewing Table 5, Panel A (for July 29, 2022), there are only 2 stocks priced at CAD 10.00 or more. Again, the third- and fourth-digit distributions with small sample sizes coupled with low predicted frequency rates cause them to breach the necessary conditions to apply the chi-square test. The leading first digit with a chi-square statistic of 6.919, coincidentally the same chi-square statistic as January 31, 2022, fits with BL. The second digit test statistic is 60.291, refuting the null hypothesis.

Table 5. For the Date July 29, 2022

Panel A: Number of Stocks with the Number by Digit Place

Number	Digit Position					
	1st digit	2nd digit	3rd digit	4th digit	5th digit	Total
0	0	84	10	1	0	95
1	194	39	3	0	0	236
2	99	41	2	0	0	142
3	90	29	3	0	0	122
4	66	28	2	0	0	96
5	54	57	8	1	0	120
6	43	28	1	0	0	72
7	47	27	5	0	0	79
8	38	26	3	0	0	67
9	36	19	3	0	0	58
Total	667	378	40	2	0	1,087

Panel B: Percentage of Number and Chi-Square Test Statistic by Digit Place

Number	Digit Position			
	1st digit	2nd digit	3rd digit	4th digit
0	N/A	22.222	25.000	50.000
1	29.085	10.317	7.500	0.000
2	14.843	10.847	5.000	0.000
3	13.493	7.672	7.500	0.000
4	9.895	7.407	5.000	0.000
5	8.096	15.079	20.000	50.000
6	6.447	7.407	2.500	0.000
7	7.046	7.143	12.500	0.000
8	5.697	6.878	7.500	0.000
9	5.397	5.026	7.500	0.000
0	N/A	22.222	25.000	50.000
Total	100.000	100.000	100.000	100.000
Chi-Square	6.919	60.291	N/A	N/A

N/A is not applicable

Lastly, Table 6, Panel A (for October 31, 2022) shows 2 stocks trading at greater than CAD 10.00. Examining, Panel B, once more the third and fourth digits cannot be evaluated by the chi-square test for the same reason as before. Monitoring the compute test statistics for the first and second digits the outcomes are 7.467 and 88.682 respectively. The first digit evidence agrees with BL as opposed to the second digit information contradicts the expected distribution as stated by BL.

Table 6. For the Date October 31, 2022

Panel A: Number of Stocks with the Number by Digit Place

	Digit Position					Total
Number	1st digit	2nd digit	3rd digit	4th digit	5th digit	
0	0	95	13	0	0	108
1	198	26	3	0	0	227
2	111	44	2	0	0	157
3	100	31	2	0	0	133
4	72	28	1	0	0	101
5	59	45	8	0	0	112
6	39	21	3	0	0	63
7	45	21	2	1	0	69
8	30	28	1	1	0	60
9	36	19	2	0	0	57
Total	690	358	37	2	0	1,087

Panel B: Percentage of Number and Chi-Square Test Statistic by Digit Place

Number	Digit Position			
	1st digit	2nd digit	3rd digit	4th digit
0	N/A	26.536	35.135	0.000
1	28.696	7.263	8.108	0.000
2	16.087	12.291	5.405	0.000
3	14.493	8.659	5.405	0.000
4	10.435	7.821	2.703	0.000
5	8.551	12.570	21.622	0.000
6	5.652	5.866	8.108	0.000
7	6.522	5.866	5.405	50.000
8	4.348	7.821	2.703	50.000
9	5.217	5.307	5.405	0.000
Total	100.000	100.000	100.000	100.000
Chi-Square	7.467	88.682	N/A	N/A

N/A is not applicable

Overall, the empirical findings were mixed with strong concurrence of the first digit, each and everyone of the 4 trading days, to BL; whereas the second digit did not, on each of the 4 occasions, follow BL. Limitations of the study occurred attributed to the low prices of stocks listed on the CSE. This situation led to theoretical values not meeting the prerequisites of the chi-square test.

6. Conclusions

Stock prices on the Canadian Securities Exchange were investigated on 4 separate dates in 2022 for compliance to that predicted by Benford’s Law; a logarithmic distribution such that lower numbers (such as 1, 2, 3) are more frequently represented than higher numbers (such as 7, 8, 9) for the leading digits. The findings supported BL for the first digit. Nevertheless, there was no empirical support for compliance with BL with the second digit. Analysis on the third and fourth digits could not be conducted for 3 of the 4 dates due to few stocks trading with 3 or 4 digits in their price. This outcome, as well as repeated examination of the stock price distribution, can be used by the CSE and investors to keep watch for systemic price manipulation on the stock exchange.

Acknowledgments

There are no acknowledgments, other than our appreciation for the support of our universities, to report for this study.

Authors' contributions

Dr. Raymond A. K. Cox designed the study, reviewed the literature, and wrote the paper. Prof. Quan Cheng gathered the data and conducted the statistical analysis. Garrett R. A. Cox explored appropriate data availabity and statistical tools for research feasibility. All authors edited the manuscript and approved the final version for submission to the International Journal of Financial Research.

Funding

No grants were secured to support this study.

Competing interests

The authors declare no conflict of interest.

Informed consent

Obtained.

Ethics approval

The Publication Ethics Committee of the Sciedu Press.

The journal and publisher adhere to the Core Practices established by the Committee on Publication Ethics (COPE).

Provenance and peer review

Not commissioned; externally double-blind peer reviewed.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

Data sharing statement

No additional data are available.

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