A Barrier Option Utility Framework for Bank Interest Margin under Government Bailout

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Abstract

The barrier options theory of corporate security valuation is applied to the contingent claims of a distressed bank under a bailout program of distressed loan purchases. In particular, the bank acts as if it has a single utility function that positively weights equity returns like, but negatively weights bankruptcy dislike. We show that an increase in the amount of distressed loan purchases decreases the loan amount at an increased margin when buying distressed loan amount is high. Bailout as such makes the bank less prone to loan risk taking, thereby contributing the stability of the banking system. A numerical exercise shows that the market-based estimates of the expected utility of bank equity returns which ignore the weights (a standard down-and-out call option) or the dislike (a standard call option) lead to significant overestimation.

Keywords: barrier option, distressed asset purchases, bank interest margin, default risk

1. Introduction

The failed bank list displayed on the Federal Deposit Insurance Corporation website shows that between January 1, 2008 and May 31, 2013, 473 banks have failed within the United States (Federal Deposit Insurance Corporation, 2013). (Note 1) The government was reluctant to see bank failures end in a straightforward liquidation since bank termination incurs bankruptcy costs. The regulatory authority affiliated by the financial crisis launched bailout programs. However, 74 banks in the United States failed to repay the government and resulted in a loss (ProPublica, 2013). (Note 2) The former failed bank list can be motivated based on a bankruptcy argument about financial stability, while the latter rescued banks can be motivated based on a profitability argument concerning bank behavior. The ongoing argument concerning the impact of government bailout on bank profits and risks warrants an assessment of the extent to which bailout affects the efficiency of bank intermediation in particular related to appropriate goal choices in modelling the bank's optimization problem including equity like and failure dislike.

Our objective is to make contributions to the literature as a result of the following expansion in methodology and scope. The original contingent claims approach to corporate security valuation views equity as a call option on the assets of the firm (Merton, 1974). This approach, assuming that the closure barrier is trivial, has found a natural application in banking intermediation (e.g., Ronn and Verma, 1986, and Episcopos, 2004). Later work has recognized that corporate securities should actually be viewed as path-dependent, barrier options (Merton, 1973, and Brockman and Turtle, 2003). Banking literature includes papers that use barrier options in some form to address the problem of early bank closure (e.g., Episcopos, 2008, and Lin et al., 2012). In regards to the method, we propose a framework for the utility function of the bank based on a weighted path-dependent, barrier option optimization algorithm instead of the commonly used nonweighted path-dependent approach. Banking is an ideal environment for the weighted barrier option utility model because the bank's manager maximizes either his/her own expected utility or that of those who exercise control over the bank's decisions of equity return like and bankruptcy dislike during a financial crisis. Bank claims are evaluated with weighted barrier options utility in the framework, providing a more suitable view of bank equity and debt.

In regard to the scope, we focus on the bank interest margin decision, i.e., the spread between the loan rate and the deposit rate, which is one of the principle elements of bank net cash flows and after-tax earnings (Lin et al., 2012). The spread is so important to bank profitability as well as to early bank closure problems (Cebula, 2010). Indeed, the bank interest margin is often used in the literature as a proxy for the efficiency of financial intermediation (Saunders and Schumacher, 2000). The purpose of the paper is to follow this argument by providing a weighted barrier option model of bank behavior to the determination of bank interest margins under the government intervention program of distressed asset purchases. (Note 3) A direct implication of this model is that bank equity will be priced as an additive, weighted down-and-out call ($WDOC$) option particularly reflecting its relative importance in the like of the bank's higher equity return and the dislike of the bank's early closure problem.

The results of this paper show that when the buying distressed assets is large, an increase in the buying amount results in a decreased loan amount held by the bank at an increased margin, thereby contributing to the banking stability. Our numerical result is supported by Hoshi and Kashyap (2010): buying distressed assets is an appropriate way to recapitalize banks. But the bailout program lacks efficiency when the purchased amount remains low. If the dislike preference is ignored, the optimal bank interest margin is positively related to the amount of distressed loan purchases when loan volatility is high perhaps due to a financial turmoil. But if the like and dislike preferences weighted by default risks are not explicitly taken into account, the optimal bank interest margin is positively related to the amount of distressed loan purchases as well. If the $WDOC$ valuation is treated as a benchmark, bank equity returns which ignore the dislike of bankruptcy problem in the call option valuation or the default risk weighted in the barrier option lead to significant overestimation. Due to the alternative goals of the bank's optimization problem for recapitalization and lending, our results should be of interest to banks, analysts and policy makers.

In related work, Gorton and Huang (2004) claim that the benefits of government bailouts depend on the type of liquidity shock faced by banks. A liquidity shock is an event where banks suddenly need new resources. Gorton and Huang (2004) find that government bailout via asset purchases are feasible, when the number of assets to be sold is too large to be absorbed by private investors. In this case, the provision of liquidity by the government increases overall welfare. In addition, Bebchuk (2008) argues that asset purchases are suitable to cope with a financial turmoil, nevertheless the author proposes a redesign of the legislation in order to achieve the targets of the program, i.e., restoring stability in the banking system, while limiting costs to taxpayers. Our paper are silent on liquidity shock in Gorton and Huang (2004) and legislation redesign in Bebchuk (2008); however, may be viewed as complementary to these studies: purchases of distressed assets by the government are effective to stabilize banks when raising new capital in public markets is difficult.

The rest of the paper is organized as follows. Section 2 develops the basic structure of the model. Section 3 derives the solution of the model and the comparative static analysis. Section 4 conducts a numerical analysis to explain the intuition of the comparative static results. The final section concludes the paper.

2. The Model

The model is designed to capture in a minimalist fashion the characteristics of a bank: (i) the bank is in distress that distressed loans are risky in that they subject to non-performance (Wong, 1997), a liquidity shock is an event that the bank suddenly needs new resources (Gorton and Huang, 2004), and the bank is more likely to face the problem of early closure (Episcopos, 2008), (ii) the bank manager likes higher equity, but dislikes higher knock-out value, implying the expected value of a utility function defined in terms the weighted like and dislike, and (iii) the weighted behavior depends on the default risk in the bank's equity returns. Note that (ii) and (iii) indicate that the utility function will have to incorporate three distinct functions: the equity of the bank is viewed as a call option on the bank's assets (Merton, 1974), the knock-out value is viewed as a down-and-in call (DIC) option (Brockman and Turtle, 2003), and the default probability is the one that the bank's assets will be less than the book value of the bank's liabilities (Brockman and Turtle, 2003). As we discuss further below, the call captures the like, the DIC reveals the dislike, and the default probability demonstrates the weight-average factor of our model.

Consider the bank that makes decisions in a single period horizon with two dates, 0 and 1, $t \in [0, 1]$. At $t = 0$, the bank has the following balance sheet:

$$(1-\theta)L + \theta L + B = D + K \tag{1}$$

where $L > 0$ is the amount of loans, $(1-\theta)L > 0$ where $0 < \theta < 1$ is the amount of non-purchased loans, $\theta L > 0$ is the purchases of loans by the government, $B > 0$ is the quantity of liquid assets, $D > 0$ is the amount of deposits, and $K > 0$ is the stock of equity capital.

We assume that the bank is a loan rate setter and loan demand is a downward-sloping function of the loan rate R_L, denoted by $L(R_L)$, $\partial L / \partial R_L < 0$, and $\partial^2 L / \partial R_L^2 < 0$ (Chen et al., 2014). $B > 0$ indicates that the bank is a net lender of the Federal funds at $t = 1$. These assets earn the security-market interest rate of $R > 0$. The bank accepts D dollars of deposits. The bank provides depositors with a rate of return equal to the risk-free rate

$R_D > 0$ (Chen et al., 2014). Equity capital held by the bank at $t = 0$ is tied by regulation to be a fixed proportion q of the bank's deposits, $K \geq qD$ where q is the required capital-to-deposits ratio (VanHoose, 2007).

The bank's objective is to set R_L to maximize the expected value of a $WDOC$ option function, subject to Eq. (1). The market value of the bank's underlying assets follows a geometric Brownian motion of the form:

$$dV = \mu V dt + \sigma V dW \tag{2}$$

and where $V = (1-\theta)(1+R_L)L$ is the bank's loan repayments, with an instantaneous drift μ, and an instantaneous volatility σ. A standard Wiener process is W. This explains that the bank's aggregate asset portfolio value obeys the assumed stochastic process. We denote by Z the book value of the net payments at $t = 0$, that has maturity equal to $t = 1$. The value of the net payments at $t = 1$ is specified as the payments to depositors $(1+R_D)D$ net of the repayments from both the liquid-asset investment $(1+R)B$ and the purchases of loans by the government. This repayment from the asset purchases is assumed to be $\theta(1+R)L$ since the government takes over the risk of illiquid loans at the price of R (Klomp, 2013). (Note 4) Z plays the role of the strike price of the standard call, since the market value of equity can be thought of as a call option on V with $t = 0$ to expiration equal to $t = 1$. The market value of equity (SC) will then be given by the Merton (1974) formula for call options:

$$SC = VN(d_1) - Ze^{-\delta}N(d_2) \tag{3}$$

where

$$Z = \frac{(1+R_D)K}{q} - (1+R)[K(\frac{1}{q}+1) - L] - \theta(1+R)L$$

$$d_1 = \frac{1}{\sigma}(\ln\frac{V}{Z} + \delta + \frac{\sigma^2}{2}),\ d_2 = d_1 - \sigma,\ \delta = (1+\theta)R - R_D$$

and where δ is the discount rate due to the specification of Z, and $N(\cdot)$ is the cumulative density function of the standard normal distribution.

Next, our approach in specifying the DIC option using Merton's (1973) model is very similar to the one used by Brockman and Turtle (2003). The value of DIC option is the difference between the SC and the down-and-out call (DOC) option where the rebate upon failure is assumed to be zero. The DIC formula represents bank bondholders (non-negative) claim and is given by

$$DIC = SC - DOC = V(\frac{H}{V})^{2\eta}N(b_1) - Ze^{-\delta}(\frac{H}{V})^{2\eta-2}N(b_2) \tag{4}$$

where

$$H = \alpha Z,\quad 0 < \alpha < 1,\quad \eta = \frac{\delta}{\sigma^2} + \frac{1}{2},\ b_1 = \frac{1}{\sigma}(\ln\frac{H^2}{VZ} + \delta + \frac{\sigma^2}{2}),\ b_2 = b_1 - \sigma$$

and where H is the value of the bank's assets that triggers bankruptcy (this is the barrier or knock-out value of the bank). For simplicity, we follow Brockman and Turtle (2003) and consider only the case of a constant barrier, $H = \alpha Z$ where $0 < \alpha < 1$ is the barrier-to-debt ratio. The DIC offers protection to bondholders by allowing them to "call in their chips" before asset values deteriorate further.

Applying Hermalin (2005), we assume the objective function can be aggregated in such a way that positively weights SC in Eq. (3), but negatively weights DIC in Eq. (4). Assume further, as in Hermalin (2005), that the objective function of the bank is additively separable:

$$S = (1-P_1)SC + (1-P_2)(-DIC) \tag{5}$$

where $(1-P_1)$ and $(1-P_2)$ are the default probability weights on the two components of like and dislike, respectively.

The first term on the right-hand side of Eq. (5) can be identified as the realized like discounted by the default risk in the bank's equity returns with the SC valuation, while the second term can be identified as the realized dislike discounted by the default risk in the bank's knock-out value with the DIC valuation. Our approach in calculating the default probability P_1 using information about Eq. (3) is outlined in Vassalou and Xing (2004). The default probability in the SC valuation is the probability that V will be less than Z. In this context, P_1 can be written as:

$$P_1 = N(-d_3) = 1 - N(d_3) \tag{6}$$

where

$$d_3 = \frac{1}{\sigma}(\ln\frac{V}{Z} + \mu - \frac{\sigma^2}{2})$$

and where d_3 is defined as the distance to default. Default occurs when the ratio of V to Z is less than 1, or its log is negative.

Next, we apply Brockman and Turtle (2003) to calculate the default probability P_2 based on Eq. (4). We arrive the SC ($=DOC$ where DIC vanishes) value that captures the bank's equity. Bankruptcy prediction is equivalent to the default probability in the DOC valuation proposed by Brockman and Turtle (2003) since the path dependency is invariant to the SC valuation. The valuation Eq. (4) implies a risk-neutral failure probability over the interval from $t \in [0,1]$ that we can write as:

$$P_2 = N(a_1) + e^{a_2}(1 - N(a_3)) \tag{7}$$

where

$$a_1 = \frac{1}{\sigma}(\ln\frac{H}{V} - \delta + \frac{\sigma^2}{2}), \quad a_2 = \frac{2}{\sigma^2}(\delta - \frac{\sigma^2}{2})\ln\frac{H}{V}, \quad a_3 = -\frac{1}{\sigma}(\ln\frac{H}{V} + \delta - \frac{\sigma^2}{2})$$

The proposed conceptual framework of Eq. (5) can be expressed mathematically in the closed-form, $WDOC$ value model. The primary feature distinguishing a DOC from a SC is the existence of a barrier which, when breached, causes the termination of the option (i.e., failure). Furthermore, the weighted-average factors are imposed on SC and DIC revealing the preference when the optimal decision is made. Our model may be viewed as complementary to the contingent claims literature.

3. Equilibrium and Comparative Static Results

The first-order condition for an optimum of Eq. (5) is:

$$\frac{\partial S}{\partial R_L} = [-\frac{\partial P_1}{\partial R_L} + (1 - P_1)\frac{\partial SC}{\partial R_L}] - (\frac{\partial P_2}{\partial R_L}DIC + P_2\frac{\partial DIC}{\partial R_L}) = 0 \tag{8}$$

where

$$\frac{\partial P_1}{\partial R_L} = -\frac{\partial N(d_3)}{\partial d_3}\frac{\partial d_3}{\partial R_L}$$

$$\frac{\partial SC}{\partial R_L} = \frac{\partial V}{\partial R_L}N(d_1) + V\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial R_L} - \frac{\partial Z}{\partial R_L}e^{-\delta}N(d_2) - Ze^{-\delta}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial R_L}$$

$$\frac{\partial P_2}{\partial R_L} = \frac{\partial N(a_1)}{\partial a_1}\frac{\partial a_1}{\partial R_L} + e^{a_2}\frac{\partial a_2}{\partial R_L}(1 - N(a_3)) - e^{a_2}\frac{\partial N(a_3)}{\partial a_3}\frac{\partial a_3}{\partial R_L}$$

$$\begin{aligned}\frac{\partial DIC}{\partial R_L} = {} & \frac{\partial V}{\partial R_L}(\frac{H}{V})^{2\eta}N(b_1) + V(2\eta)(\frac{H}{V})^{2\eta}(\frac{1}{H}\frac{\partial H}{\partial R_L} - \frac{1}{V}\frac{\partial V}{\partial R_L})N(b_1) \\ & + V(\frac{H}{V})^{2\eta}\frac{\partial N(b_1)}{\partial b_1}\frac{\partial b_1}{\partial R_L} \\ & - \frac{\partial Z}{\partial R_L}e^{-\delta}(\frac{H}{V})^{2\eta-2}N(b_2) - Ze^{-\delta}(2\eta - 2)(\frac{H}{V})^{2\eta-2}(\frac{1}{H}\frac{\partial H}{\partial R_L} - \frac{1}{V}\frac{\partial V}{\partial R_L})N(b_2) \\ & - Ze^{-\delta}(\frac{H}{V})^{2\eta-2}\frac{\partial N(b_2)}{\partial b_2}\frac{\partial b_2}{\partial R_L}\end{aligned}$$

We require that the second-order condition be satisfied, that is $\partial^2 S / \partial R_L^2 < 0$. The first term $[\cdot]$ on the right-hand ride of Eq. (8) can be interpreted as the marginal equity value of loan rate in the weighted SC valuation, while the second term $(\cdot)$ can be interpreted as the marginal barrier value of loan rate in the weighted DIC valuation. The optimal bank interest margin is set by the bank where both the weighted marginal values are equal.

Consider next the impact on the bank's loan rate (and thus on the bank's interest margin) from changes in distressed loan purchases. Implicit differentiation of Eq. (8) with respect to θ yields:

$$\frac{\partial R_L}{\partial \theta} = -\frac{\partial^2 S}{\partial R_L \partial \theta} / \frac{\partial^2 S}{\partial R_L^2} \tag{9}$$

where

$$\frac{\partial^2 S}{\partial R_L \partial \theta} = [-\frac{\partial^2 P_1}{\partial R_L \partial \theta} - \frac{\partial P_1}{\partial \theta}\frac{\partial SC}{\partial R_L} + (1-P_1)\frac{\partial^2 SC}{\partial R_L \partial \theta}]$$
$$-[\frac{\partial^2 P_2}{\partial R_L \partial \theta} DIC + \frac{\partial P_2}{\partial R_L}\frac{\partial DIC}{\partial \theta} + \frac{\partial P_2}{\partial \theta}\frac{\partial DIC}{\partial R_L} + P_2 \frac{\partial^2 DIC}{\partial R_L \partial \theta}]$$

The sign of Eq. (9) is governed by its numerator since the second-order condition is limited to $\partial^2 S / \partial R_L^2 < 0$. In general, the added complexity of a weighted barrier-based option does not always lead to clear-cut results in the current form of Eq. (9). However, we can certainly speak of tendencies for reasonable parameter levels corresponding roughly to Eq. (9) with the bank's equity value in the $WDOC$ valuation. The numerical examples provide intuition regarding problems at hand, for example, the comparative static results of Eq. (9).

4. Numerical Exercises

Unless otherwise indicated, the parameter values are $R = 3.5\%$, $R_D = 2.5\%$, $q = 9.0\%$, $\alpha = 0.2$, and $\mu = 0.2$. Let $(R_L\%, L)$ change from (4.1, 300) to (4.7, 279) due to the conditions of $\partial L / \partial R_L < 0$ and $\partial^2 L / \partial R_L^2 < 0$. The value of θ is assumed to increase from 0.1 to 0.7 due to the condition of $0 < \theta < 1$. Note that (i) $R_L > R_D$ demonstrates that the bank interest margin is used as a proxy for the efficiency of financial intermediation (Lin et al., 2012), (ii) $R_L > R$ indicates that there is asset substitution of the earning-asset portfolio (Kashyap et al., 2002), (iii) R is sufficiently larger than R_D when the capital requirement constraint is binding (Wong, 1997), and (iv) $q = 9.0\%$ implies that the specification of capital adequacy requirement is consistent with the approach of the Basel (VanHoose, 2007). The numerical parameters presented above can be intuitively interpreted as being closely approaching a real state of a hypothetical bank.

Before proceeding with the analysis of the comparative static results in Eq. (9), first of all, we observe equity components of bank expected values including SC in Eq. (3), $DOC = SC - DIC$ where DIC is defined as Eq. (4), and S in Eq. (5). The two examples of $\theta = 0.3$ and 0.6 are limited to $0.1 \le \theta \le 0.7$ in our numerical exercises, as mentioned earlier. The possible conditions of $\sigma = 0.3$ and 0.6 and $\alpha = 0.2$ are due to Brockman and Turtle (2003). (Note 5) The findings are summarized in Table 1.

Table 1. Equity components of bank expected value [*]

θ	(R_L%, L) (4.1, 300)	(4.2, 299)	(4.3, 297)	(4.4, 294)	(4.5, 290)	(4.6, 285)	(4.7, 279)
	SC ($\sigma = 0.3$)						
0.3	43.8699	43.9433	43.9305	43.8308	43.6433	43.3677	43.0036
0.6	33.7877	33.8525	33.8780	33.8641	33.8108	33.7186	33.5882
	DOC ($\sigma = 0.3$)						
0.3	38.1834	38.2458	38.2279	38.1293	37.9498	37.6894	37.3485
0.6	29.0627	29.1284	29.1613	29.1616	29.1297	29.0666	28.9736
	S ($\sigma = 0.3$)						
0.3	31.5081	31.6094	31.6610	31.6623	31.6135	31.5146	31.3666
0.6	26.4759	26.5682	26.6468	26.7117	26.7633	26.8025	26.8304
	SC ($\sigma = 0.6$)						
0.3	65.7234	65.7002	65.5023	65.1286	64.5782	63.8500	62.9433
0.6	43.9403	43.9407	43.8474	43.6600	43.3784	43.0026	42.5330
	DOC ($\sigma = 0.6$)						
0.3	60.5396	60.5069	60.3046	59.9319	59.3883	58.6733	57.7867
0.6	39.5869	39.5881	39.5011	39.3260	39.0629	38.7122	38.2750
	S ($\sigma = 0.6$)						
0.3	36.2921	36.3274	36.2792	36.1470	35.9306	35.6300	35.2455
0.6	26.5671	26.6148	26.6269	26.6036	26.5457	26.4544	26.3316

[*] Expected values of SC in Eq. (3), $DOC = SC - DIC$ where DIC is defined as Eq. (4), and S in Eq. (5) are computed using the following parameter values, unless indicated otherwise, $R = 3.5\%$, $R_D = 2.5\%$, $q = 9.0\%$, $\alpha = 0.2$, and $\mu = 0.2$.

Several results are observed from Table 1. (i) The equity value in the SC valuation is consistently larger than the equity value in the DOC valuation, implying that the knock-out value of the bank (the DIC call based on Eq. (4)) is positive in sign. Our finding is supported by the empirical evidence of Brockman and Turtle (2003). (ii) We show that the equity value in the DOC valuation is consistently larger than the equity value in the $WDOC$ valuation in Eq. (5), implying that the effects of default risk on equity return in the SC valuation and on knock-out value in the DIC valuation are significant. The gap value between DOC and $WDOC$ (S in Eq. (5)) indicates that the bank may have incentives to make decisions explicitly taking into account the like of equity and the dislike of knock-out value. (iii) It is interesting that, as volatility of the underlying asset increases, both the values in the SC and DOC valuation are increased. But, asset volatility has an ambiguous effect on a $WDOC$ option, in general. The reason is that there are two weighted effects in play. Increased volatility means that the weighted barrier is more likely be breached (thus, canceling the option), but leads to higher expected weighted payoff if the weighted barrier is not breached.

Table 2. Values of S and $\partial R_L / \partial\theta$ when $\sigma = 0.3$ *

θ	$(R_L\%, L)$ (4.1, 300)	(4.2, 299)	(4.3, 297)	(4.4, 294)	(4.5, 290)	(4.6, 285)	(4.7, 279)
	S						
0.1	36.0057	36.1121	36.1469	36.1094	35.9992	35.8159	35.5599
0.2	33.6951	33.7989	33.8419	33.8234	33.7433	33.6015	33.3985
0.3	31.5081	31.6094	31.6610	31.6623	31.6135	31.5146	31.3666
0.4	29.5017	29.6005	29.6611	29.6832	29.6670	29.6129	29.5220
0.5	27.7704	27.8666	27.9365	27.9801	27.9979	27.9904	27.9589
0.6	26.4759	26.5682	26.6468	26.7117	26.7633	26.8025	26.8304
0.7	25.8778	25.9622	26.0447	26.1250	26.2034	26.2803	26.3563
	$\partial^2 S / \partial R_L \partial\theta$						
0.1→0.2	-0.0013	0.0041	0.0095	0.0151	0.0207	0.0265	
0.2→0.3	-0.0012	0.0043	0.0099	0.0156	0.0215	0.0275	
0.3→0.4	-0.0012	0.0045	0.0104	0.0163	0.0224	0.0286	
0.4→0.5	-0.0014	0.0047	0.0108	0.0170	0.0233	0.0297	
0.5→0.6	-0.0019	0.0043	0.0106	0.0170	0.0233	0.0297	
0.6→0.7	-0.0039	0.0019	0.0077	0.0134	0.0188	0.0241	
	$\partial^2 S / \partial R_L^2$						
0.1	-	-1.7897	-1.8077	-1.8194	-1.8248	-1.8177	-
0.2	-	-1.5216	-1.5354	-1.5422	-1.5412	-1.5293	-
0.3	-	-1.2449	-1.2544	-1.2560	-1.2497	-1.2288	-
0.4	-	-0.9569	-0.9621	-0.9586	-0.9456	-0.9200	-
0.5	-	-0.6552	-0.6567	-0.6486	-0.6294	-0.6012	-
0.6	-	-0.3426	-0.3420	-0.3320	-0.3109	-0.2827	-
0.7	-	-0.0499	-0.0525	-0.0484	-0.0375	-0.0217	-
	$\partial R_L / \partial\theta$, ‰						
0.1→0.2	-	-0.0072	0.0225	0.0523	0.0825	0.1141	-
0.2→0.3	-	-0.0082	0.0279	0.0643	0.1014	0.1403	-
0.3→0.4	-	-0.0099	0.0360	0.0825	0.1306	0.1823	-
0.4→0.5	-	-0.0142	0.0486	0.1125	0.1796	0.2533	-
0.5→0.6	-	-0.0293	0.0660	0.1638	0.2695	0.3881	-
0.6→0.7	-	-0.1149	0.0561	0.2322	0.4303	0.6667	-

* The value of equity in Eq. (5) is computed using the following parameter values, unless indicated otherwise, $R = 3.5\%$, $R_D = 2.5\%$, $q = 9.0\%$, $\alpha = 0.2$, and $\mu = 0.2$. Shaded area in the first panel represents an approximate equity value with a corresponding optimal loan rate.

In the first panel of Table 2, we have the result of $S > 0$. Note that shaded areas represent, for example, the approximate maximum equity value of 36.1469 with the corresponding optimal loan rate of 4.3%, and that of 31.6623 with the corresponding optimal loan rate of 4.4%. Correspondingly, we have the results of $\partial^2 S / \partial R_L \partial\theta > 0$ observed from the shaded areas in the second panel. $\partial^2 S / \partial R_L^2 < 0$ presented in the third panel explains the validness of the second-order condition. Accordingly, we have the result of $\partial R_L / \partial\theta > 0$ observed from the shaded areas in the last panel that the bank's interest margin is increased as the amount of the distressed loan purchases increases. Intuitively, as the bank participates in the program of distressed loan purchases to decrease its distressed loans holding, it now provides an expected return to a less risk base. One way the bank may attempt to augment its total returns is by shifting its investments to the liquid-asset market and away from its loan portfolio. If loan demand is relatively rate-elastic, a less loan portfolio is possible at an increased margin. Government intervention as such makes the bank less prone to risk-taking, thereby contributing the stability of the banking system. We argue that buying distressed assets in the bailout program is an appropriate way to recapitalize banks, which is consistent with the empirical findings of Hoshi and Kashyap (2010).

Table 3. Values of $\partial R_L / \partial \theta$ at various levels of σ in the S valuation *

	σ			
θ	0.3	0.4	0.5	0.6
	S (R_L%)			
0.1	36.1469 (4.3)	38.8459 (4.2)	41.3572 (4.2)	43.5960 (4.2)
0.2	33.8419 (4.3)	36.0030 (4.3)	38.0620 (4.2)	39.9248 (4.2)
0.3	31.6623 (4.4)	33.2801 (4.3)	34.8561 (4.2)	36.3274 (4.2)
0.4	29.6832 (4.4)	30.7207 (4.4)	31.7877 (4.2)	32.8445 (4.2)
0.5	27.9979 (4.5)	28.4280 (4.4)	28.9581 (4.3)	29.5530 (4.2)
0.6	26.8304 (4.7)	26.6286 (4.5)	26.5485 (4.4)	26.6269 (4.3)
	$\partial R_L / \partial \theta$, ‰			
0.1→0.2	0.0225	-0.0010	0.0020	0.0037
0.2→0.3	0.0279	0.0327	0.0031	0.0050
0.3→0.4	0.0825	0.0424	0.0052	0.0072
0.4→0.5	0.1125	0.0592	0.0093	0.0114
0.5→0.6	0.2695	0.1808	0.0918	0.0270

* The value in Eq. (5) is computed using the following parameter values, unless indicated otherwise, $R = 3.5\%$, $R_D = 2.5\%$, $q = 9.0\%$, $\alpha = 0.2$, and $\mu = 0.2$. The qualitative results of $\partial^2 S / \partial R_L^2$ in the cases of $\sigma = 0.4$, 0.5, and 0.6 are negative in sign that confirm their validness of the second-order condition. The computation of $\sigma = 0.4$, 0.5, and 0.6 follows a similar one as in the case of $\sigma = 0.3$ in Table 2. Note that S (R_L%) represents the approximate equity value with a corresponding optimal loan rate in the $WDOC$ valuation.

Table 3 presents the following results. (i) An increase in the asset volatility increases the bank's equity evaluated at the optimal loan rate when the amount of the distressed loan purchases is low, and has an indeterminate effect on the bank's equity when the purchase amount is high observed from the upper panel of Table 3. With a government as the lender of last resort in particular when buying distressed assets remains low, there is an incentive for the bank to increase the credit risk profile in order to obtain a higher expected payoff for shareholders (Breitenfellner and Wagner, 2010). (ii) The higher the asset volatility, the lower likely bank interest margin becomes, which is consistent with the empirical findings of Williams (2007). (iii) When the distressed asset amount purchased is high, an increase in the amount of the distressed loan purchases increases the bank's interest margin. Government intervention as such makes the bank less prone to risk-taking, which is consistent with Hoshi and Kashyap (2010). (iv) The positive impact on the bank's interest margin from increases in the relatively large amount of the distressed loan purchases is less significant when bank loan variability becomes larger.

Table 4. Values of $\partial R_L / \partial \theta$ at various levels of σ in the SC valuation *

	σ			
θ	0.3	0.4	0.5	0.6
	SC (R_L%)			
0.1	51.3927 (4.2)	61.0103 (4.2)	70.8064 (4.1)	80.6523 (4.1)
0.2	47.6310 (4.2)	55.9795 (4.2)	64.5376 (4.2)	73.1684 (4.1)
0.3	43.9433 (4.2)	51.0066 (4.2)	58.3175 (4.2)	65.7234 (4.1)
0.4	40.3658 (4.2)	46.1213 (4.2)	52.1688 (4.2)	58.3379 (4.1)
0.5	36.9742 (4.3)	41.3776 (4.2)	46.1370 (4.2)	51.0503 (4.1)
0.6	33.8780 (4.3)	36.8834 (4.2)	40.3151 (4.2)	43.9407 (4.2)
	$\partial R_L / \partial \theta$, ‰			
0.1→0.2	-0.0060	0.0002	-	-
0.2→0.3	-0.0065	0.0005	0.0045	-
0.3→0.4	-0.0072	0.0010	0.0055	-
0.4→0.5	-0.0080	0.0021	0.0072	-
0.5→0.6	0.0485	0.0041	0.0103	0.0210

* SC based on Eq. (3) is computed using the following parameter values, unless indicated otherwise, $R = 3.5\%$, $R_D = 2.5\%$, $q = 9.0\%$, $\alpha = 0.2$, and $\mu = 0.2$. SC (R_L%) represents an approximate equity value with a corresponding optimal loan rate. $\partial^2 SC / \partial R_L^2 < 0$ in the four cases ($\sigma = 0.3$, 0.4, 0.5, and 0.6) confirms their validness of the second-order condition. The computation of $\partial R_L / \partial \theta = -(\partial^2 SC / \partial R_L \partial \theta) / (\partial^2 SC / \partial R_L^2)$ follows a similar one as in the case of the S valuation in Table 2.

Alternatively, suppose that barrier is not in effect. The findings are summarized in Table 4. (i) For a given level of buying distressed loans, the equity return with a negatively corresponding optimal margin is positively related to loan volatility. This result is understood that there is an incentive for the bank to increase the risk profile in order to obtain a higher expected equity return at a given level of bailout (Breitenfellner and Wagner, 2010). (ii) For a given level of asset variability, the equity return with a positively corresponding optimal margin is negatively related to the bailout amount of buying distressed loans. This explains that the bailout program is costly to the distressed bank, which is consistent with the empirical findings of Hoshi and Kashyap (2010): the total amount of assets purchased remains low in the Troubled Asset Relief Program in the United States. (iii) We observe the results from the lower panel in Table 4 and find $\partial R_L / \partial\theta > 0$ when loan volatility is high. Bailout as such makes the bank less prone loan risk taking at an increased margin, thereby contributing the stability in the banking system. This result is consistent with the findings of Hoshi and Kashyap (2010): buying distressed assets from banks is an appropriate way to recapitalize banks. Therefore, quantifying the role of the underlying asset volatility on bank contingent claims is important.

Table 5. Values of $\partial R_L / \partial\theta$ at various levels of σ in the DOC valuation *

θ	σ 0.3	0.4	0.5	0.6
	DOC (R_L%)			
0.1	45.2804 (4.2)	55.2252 (4.2)	65.1895 (4.1)	75.1173 (4.1)
0.2	41.7136 (4.2)	50.3736 (4.2)	59.0874 (4.1)	67.7981 (4.1)
0.3	38.2458 (4.2)	45.6032 (4.2)	53.0543 (4.1)	60.5396 (4.1)
0.4	34.9248 (4.2)	40.9541 (4.2)	47.1248 (4.1)	53.3722 (4.1)
0.5	31.8490 (4.3)	36.4982 (4.2)	41.3727 (4.2)	46.3512 (4.1)
0.6	29.1616 (4.4)	32.3759 (4.2)	35.9079 (4.2)	39.5881 (4.2)
	$\partial R_L / \partial\theta$, ‰			
0.1→0.2	0.0013	0.0050	-	-
0.2→0.3	0.0015	0.0063	-	-
0.3→0.4	0.0022	0.0079	-	-
0.4→0.5	0.0034	0.0100	-	-
0.5→0.6	0.0053	0.0143	0.0182	-

* $DOC = SC - DIC$ based on Eqs. (3) and (4) is computed using the following parameter values, unless indicated otherwise, $R = 3.5\%$, $R_D = 2.5\%$, $q = 9.0\%$, $\alpha = 0.2$, and $\mu = 0.2$. DOC (R_L%) represents an approximate equity value with a corresponding optimal loan rate. $\partial^2 DOC / \partial R_L^2 < 0$ in the four cases ($\sigma = 0.3$, 0.4, 0.5, and 0.6) confirms their validness of the second-order condition. The computation of $\partial R_L / \partial\theta = -(\partial^2 DOC / \partial R_L \partial\theta) / (\partial^2 DOC / \partial R_L^2)$ follows a similar one as in the case of the S valuation in Table 2.

Next, suppose that the default risk weighted in the $WDOC$ option valuation is not in effect. Under this view, the comparative static results of $\partial R_L / \partial\theta$ at various levels of asset volatility are summarized in Table 5. We show that an increase in the amount of buying distressed loans reduces the loan portfolio held by the bank at an increased margin. Basically, increases in the amount of distressed loan purchases encourage the bank to shift investments to the liquid-asset market, the bank increase the size of its spread in order to reduce the amount of loans.

In the following subsection, we use the results presented in Tables 3, 4, and 5 to compare the effects of distressed loan purchases on the bank's volatility under the $WDOC$ with those under the SC or the DOC. The results are as follows. (i) An increase in the loan volatility increases the bank's equity return with a negatively corresponding optimal bank interest margin, except that the distressed loan purchases remain high in the $WDOC$ valuation. (ii) If the benchmark is treated as the $WDOC$ valuation, we find that market-based estimates of bank equity returns in the SC and DOC valuations lead to significant overestimations. (iii) The impact on bank interest margin from changes in buying distressed loans have a positive effect in the DOC valuation, in the $WDOC$ valuation when purchasing amount remains large, and in the SC valuation when loan volatility is high. Our results are consistent with Santomero (1984) that the choice of an appropriate goal in modeling the bank's optimization problem remains a controversial issue. These results based on different objectives may be also due to the conflicting goals of the program of distressed asset purchases for bank recapitalization and bank lending.

5. Conclusions

The paper proposes a barrier option utility approach for bank interest margin determination under distressed asset purchases by the government based on an application of the barrier option framework of Brockman and Turtle (2003). Specifically, the objective function includes the like of higher equity return based on the weighted standard call option and the dislike of higher knock-out value based on the weighted DIC option. We show that the bailout program of distressed asset purchases is appropriate. One issue that has not been addressed is the optimal bank interest margin management under a design of rescue packages. For example, the Troubled Asset Relief Program is a combination of equity injections and distressed asset purchases, while most European bailout programs combine government guaranteed debt issuance programs with direct equity injections (Breitenfellner and Wagner, 2010). In particular, is it the case that the results of this paper also apply to rescue package alternatives. The aforementioned issue may provide a maple opportunity for future research.

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Notes

Note 1. Out of this figure, 140 of these banks failed in 2009, while 156 failed in 2010 (Federal Deposit Insurance Corporation, 2013).

Note 2. These transactions are final and will never result in a profit for taxpayers (ProPublica, 2013).

Note 3. Among the possible means of government intervention are government guaranteed debt issuance programs, direct equity injections, and purchases of distressed assets by the government. Our analysis is limited to the last one. Results to be derived from our model may not extend to the cases where the bank is bailed out by the debt insurance and equity injection programs (see Breitenfellner and Wagner, 2010).

Note 4. Asset purchases may even provide capital relief, if purchases are higher than book values. However, the purchase of assets at prices below book value would instead imply a forced write-down and a fire sale for the recipient institution (Klomp, 2013). Our model focuses on the cost of risk mitigation at the price of the security-market interest rate since $\theta(1+R_L)L$ becomes a risk-free repayment with government bailout.

Note 5. The average asset volatility is 0.2904 with a corresponding standard deviation of 0.2608, and the average barrier estimate by debt load ($0.1 < \text{debt proportion} \le 0.2$) is 0.4151 with a corresponding standard deviation of 0.1529 (Brockman and Turtle, 2003).