ORIGINAL RESEARCH

A recurrence network approach for the analysis of skin blood flow dynamics in response to loading pressure

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Abstract

This paper presents a recurrence network approach for the analysis of skin blood flow dynamics in response to loading pressure. Recurrence is a fundamental property of many dynamical systems, which can be explored in phase spaces constructed from observational time series. A visualization tool of recurrence analysis called recurrence plot (RP) has been proved to be highly effective to detect transitions in the dynamics of the system. However, it was found that delay embedding can produce spurious structures in RPs. Network-based concepts have been applied for the analysis of nonlinear time series recently. We demonstrate that time series with different types of dynamics exhibit distinct global clustering coefficients and distributions of local clustering coefficients and that the global clustering coefficient is robust to the embedding parameters. We applied the approach to study skin blood flow oscillations (BFO) response to loading pressure. The results showed that global clustering coefficients of BFO significantly decreased in response to loading pressure ($p<0.01$). Moreover, surrogate tests indicated that such a decrease was associated with a loss of nonlinearity of BFO. Our results suggest that the recurrence network approach can practically quantify the nonlinear dynamics of BFO.

Key words

Blood flow oscillations, Laser doppler flowmetry, Recurrence quantification analysis, Pressure ulcers

1 Introduction

Pressure ulcers are a common but potentially preventable condition in people with physical disabilities [1]. Although the etiology of pressure ulcers has remained unclear, prolonged tissue ischemia has been cited as the most important mechanism [2]. Tissue ischemia occurs when externally applied pressure causes the occlusion of blood vessels, thus reducing blood flow to local tissues [3]. It has been demonstrated that sustained pressure causes not only a decrease in skin blood flow, but also alterations in blood flow oscillations (BFO) [4, 5]. However, the relationship between the alterations in BFO and tissue ischemia is poorly understood.

Skin BFO have been extensively studied by means of laser Doppler flowmetry (LDF) for the assessment of microcirculatory function. Spectral analysis of LDF signals has revealed five characteristic frequencies associated with metabolic, neurogenic, myogenic, respiratory, and cardiac frequencies, respectively [6]. Usually the power of BFO within each frequency interval is used as a quantitative measure for characterizing the states of the underlying physiological mechanisms [6, 7]. Based on wavelet analysis, Li et al. [4] showed that pronged compression attenuated the endothelial related metabolic activities. Jan et al. [8] showed that blood flow control mechanisms exhibited different responses to alternating pressure and constant pressure. These studies, however, did not provide insights regarding the dynamics of BFO in response to pressure.

The dynamics of BFO can be characterized using nonlinear measures [9, 10]. Most popular nonlinear measures (e.g. Lyapunov exponents, correlation dimension, and entropies) have a common feature of quantifying certain invariant properties of phase space. These measures generally require long and stationary data series. Another important method for describing complex dynamics is recurrence plots (RPs) [11]. A RP is a graphical representation of recurrences in phase space. Since recurrence is a fundamental property of dynamic systems, RPs derived from experimental time series often exhibit certain structures, e.g. diagonal or vertical lines formed by recurrence points. A variety of statistical measures based on the length distributions of these lines have been suggested to characterize different aspects of dynamic complexity of the studied system. This conceptual framework is known as recurrence quantification analysis (RQA) and has found numerous applications [12-14]. An advantage of RQA is that it is even useful in analyzing short and non-stationary data. However, it was found that delay embedding produces spurious structures in RPs [15].

Recently, the success of network theory in various scientific disciplines has motivated attempts to apply this concept to the analysis of time series [16, 17]. Zhang and Small [16] first suggested to study time series from the complex network perspective. In their scheme, each cycle in the time series is represented by a node of the network and those nodes are connected if their corresponding cycles are morphologically similar. In this way, the dynamics of time series are encoded into the topology of the network, which is then quantified by the statistical properties of the network. Lacasa et al. [18] proposed an alternative method, called visibility graphs, that converts a time series into a graph. In the graph, every node corresponds, in the same order, to series data. Any two data values are considered as two connected nodes of the graph if a constraint on their relative magnitudes is fulfilled. Yang and Yang [19] proposed to construct complex networks from the correlation matrix of a time series. By embedding a time series, state vectors in the phase space are considered as vertices of a complex network. Two vertices are connected if the Pearson correlation coefficient of the corresponding state vectors is larger than a given threshold. From the point view of time series analysis, these methods suffer from specific limitations [17].

In this paper, we present a new conceptual approach, the recurrence network approach, for analyzing the structural properties of skin blood flow signals. This approach interprets the recurrence matrix of a time series as the adjacency matrix of a network. Donner et al. [17] suggested that the measures of recurrence networks can capture the higher-order statistical properties of the invariant density of the dynamical system captured in an observational time series. Such properties are not yet provided by the existing methods of time series analysis. However, this approach has only been applied to low-dimensional model systems as well as a few real data [17, 20, 21].

The remainder of this paper is organized as follows. In section 2, we briefly describe RQA. Then we show that a fundamental measure of RQA is sensitive to the embedding parameters. In section 3, we discuss a measure of recurrence networks, clustering coefficient, and demonstrate that different types of dynamics lead to distinct distributions of local clustering coefficients. We also demonstrate that global clustering coefficient is robust to the embedding parameters. Section 4 presents the results of the approach for skin blood flow data. Finally, section 5 gives our conclusion.

2 Recurrence quantification analysis (RQA)

RP was introduced by Eckmann et al. [11] in the late 1980's to visualize the recurrences of dynamical systems. Given a time series $\{x(i),\ i = 1, \dots, M\}$, choosing a time delay τ and an embedding dimension m, one constructs a phase space trajectory

$$\mathrm{x}_i = (x(i), x(i+\tau), \ldots, x(i+(m-1)\tau), \quad i = 1, \ldots, N = M-(m-1)\tau. \tag{1}$$

A RP is a representation of the recurrence matrix

$$R_{ij} = \Theta\left(\varepsilon - d\left(\mathrm{x}_i - \mathrm{x}_j\right)\right), \tag{2}$$

where $\Theta(\cdot)$ is the Heaviside function, d is the distance between x_i and x_j, and ε is a predefined threshold. Using a spatial distance as the recurrence criterion, the matrix R is symmetric with R_{ij}=1 if the state x_j is a neighbor of x_i in phase space, and R_{ij}=0 otherwise.

To see how a RP represents the dynamics of a dynamical system, let us consider the RPs of three typical time series, e.g. a sine wave, one realization of the Lorenz system uniformly distributed white noise, and a skin blood flow signal (see Figure 1). The Lorenz system is defined as

$$dx/dt = \sigma(y-x)$$

$$dy/dt = x(r-y)$$

$$\frac{dz}{dt} = xy - \beta z \tag{3}$$

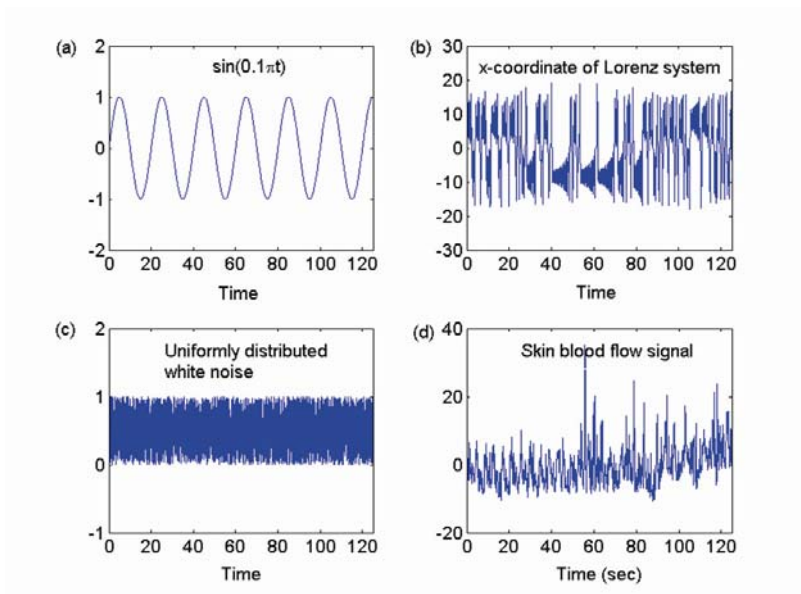

Figure 1. (a) A sine wave x=sin(0.1π*t*) with sampling frequency 32 Hz. (b) The *x*-component of the Lorenz system (equation 3) with the parameters σ=10, r=28, β=8/3, sampling time Δt=0.03, and initial condition [0.1, 0, 0]. (c) Uniformly distributed white noise, supposing that the sampling frequency is 32 Hz. (d) A skin blood flow signal recorded at a sampling frequency of 32 Hz.

The embedding parameters are separately selected for each time series. The time delay τ is selected as the first minimum of the auto mutual information function, and the embedding dimension m is determined using the Cao's method [22]. The recurrence threshold ε is selected such that the global recurrence rate

$$RR = \frac{1}{N^2}\sum_{i,j=1}^{N} R_{i,j} \tag{4}$$

is fixed at the same value [20]. As shown in Figure 2, the RPs of time series with different types of dynamics exhibit distinct structural properties. The RP of the sine wave is characterized by long and non-interrupted diagonal lines. The vertical distance between these lines corresponds to the period of the signal. The RPs of the Lorenz system and the blood flow signal also exhibits diagonals but they are not as regular as in the case of the sine wave, and their lengths are obviously shorter. For uniformly distributed white noise, the RP mainly consists of isolated black points.

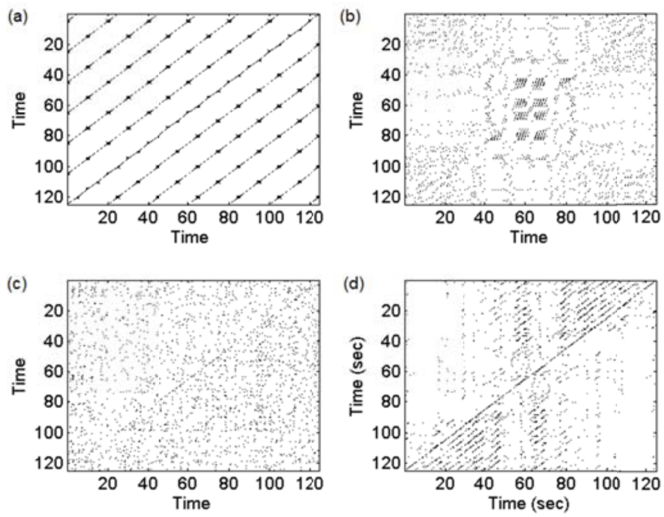

Figure 2. (a) RP of the sine wave (Figure 1a) with parameters m=3, τ=6. The distance between the diagonal lines equals to the period. (b) RP of the x-component of the Lorenz system (Figure 1b) with m=3, τ=5. (c) RP of uniformly distributed white noise (Figure 1c) with m=7, τ=1. (d) RP of the skin blood flow signal (Figure 1d) with m=7, τ=30. For each series, the threshold ε was selected preserving RR=0.03 (equation 2).

The study of recurrences by means of RPs, known as recurrence quantification analysis (RQA), is based on the recurrence point density and the diagonal and vertical line structures [23]. The lengths of diagonal lines are related to the predictability of the time series, and the presence of vertical lines indicates slowly changing states. Here we consider only the structures of diagonal lines. A diagonal line of length l indicates that two segments of the trajectory are close to each other for l time steps. Thus, the ratio

$$DET = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)} \tag{5}$$

has been adopted as a measure of determinism or predictability of the system [23], where $P(l)$ is the frequency distribution of the length l, and l_{min} is the length threshold. The use of l_{min} is to exclude the diagonal lines which are formed due to tangential motion of the phase space trajectory [23]. For l_{min} =1, the determinism is one. Marwan et al. [23] suggested that the choice of l_{min} could be made in the similar way as the choice of the size of Theiler window [24].

The measure DET involves four parameters: time delayτ, embedding dimension$m$, distance threshold ε, and minimal length of diagonal lines l_{min}. Thiel et al. [15] showed that delay embedding procedure can cause spurious correlations between the distances of embedded vectors in phase space and thus produces spurious structures in the RP [15]. Here we demonstrate that the determinism measure DET is rather sensitive to the embedding parameters. We calculated DET of the x-component of the Lorenz system (see Figure 1b) and the blood flow signal (see Figure 1d) for varying values of τ and m around their optimal values. For the Lorenz system, the optimal values of τ and m are 5 and 3 respectively; for the blood flow signal, the optimal values of τ and m are 30 and 7 respectively. Thus, we calculated DET of the Lorenz system for m=1~10 while τ=5 and τ=1~10 while m=3. DET of the blood flow signal was calculated for m=1~15 while τ=7 and τ=10~50 while m=3. For both signals, the parameter l_{min}=4 was used, and the threshold ε was selected preserving RR=0.03. The results showed that DET is sensitive to the embedding parameters (see Figure 3).

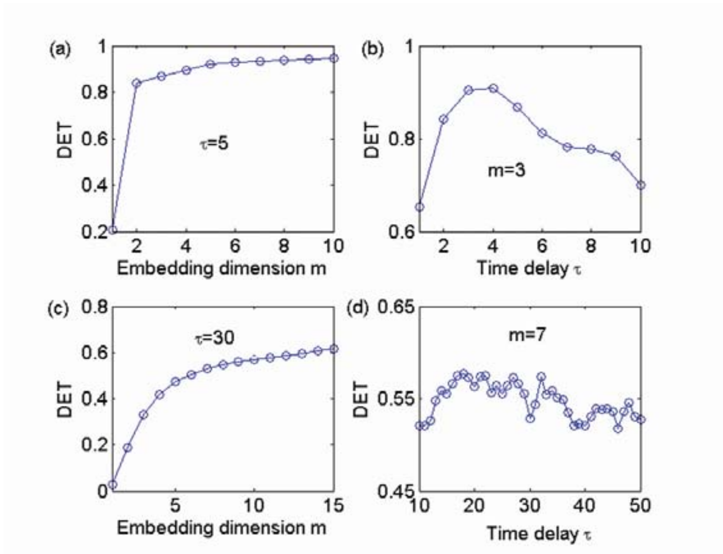

Figure 3. Influence of time delay τ and embedding dimension m on the RQA measure DET (equation 5) for (a)(b) the x-component of the Lorenz system (see Figure 1b) and (c)(d) the blood flow signal (Figure 1d). For the Lorenz system, the optimal values of τ and m are 5 and 3, respectively; for the blood flow signal, the optimal values of τ and m are 30 and 7, respectively. For each series, threshold ε was selected preserving RR=0.03 and l_min=4 was used.

3 Recurrence network approach

A new conceptual approach for analyzing structural features of time series is to consider the phase space vectors as nodes of an undirected and unweighted network and quantify the topological features of the network [17]. For this purpose, we define an adjacency matrix

$$A_{ij} = R_{ij} - \delta_{ij} \tag{6}$$

where δ_{ij} is the Kronecker delta for excluding the line of identity from the RP. The adjacency matrix and the recurrence matrix exhibit strong analogy: an adjacency matrix represents connections in a network and a recurrence matrix represents neighbored state vectors in phase space. This approach on creating complex network is more natural and simple than the other methods. To illustrate the potential of the recurrence network approach for characterizing the dynamic properties of BFO, we focus on the local clustering coefficient of vertex x_i [25]

$$C_i = \sum_{k>j} A_{ij}A_{ik}\,A_{jk} / \sum_{k>j} A_{ij}A_{ik} \tag{7}$$

where $\sum_{k>j} A_{ij}A_{ik}\,A_{jk}$ is the number of triangles involving vertex i, and $\sum_{k>j} A_{ij}A_{ik}$ is the number of connected triples with vertex i being the central vertex. A triangle is a set of three vertices where each vertex is adjacent to other two vertices. A connected triple is also a set of three vertices where two vertices are adjacent to the central vertex. Note that if vertex i has no or only one neighbor, i.e. $\sum_{k>j} A_{ij}A_{ik}$=0, C_i is defined as C_i=0 [17]. Thus, C_i gives the probability that two neighbors (recurrences) of the state x_i are also neighbors. Donner et al. [17] suggested that C_i characterizes higher order properties related to the heterogeneity of the phase space density in the vicinity of the vertex x_i. For chaotic systems high values of C_i often coincide with dynamically invariant objects such as unstable periodic orbits [20]. When a trajectory visits the neighborhood of an unstable periodic orbit, it falls into this neighborhood for a certain duration, during which the probability of recurrences is increased and local divergence rate becomes smaller. The global clustering coefficient is defined as

$$C = \sum_i C_i / N \ . \tag{8}$$

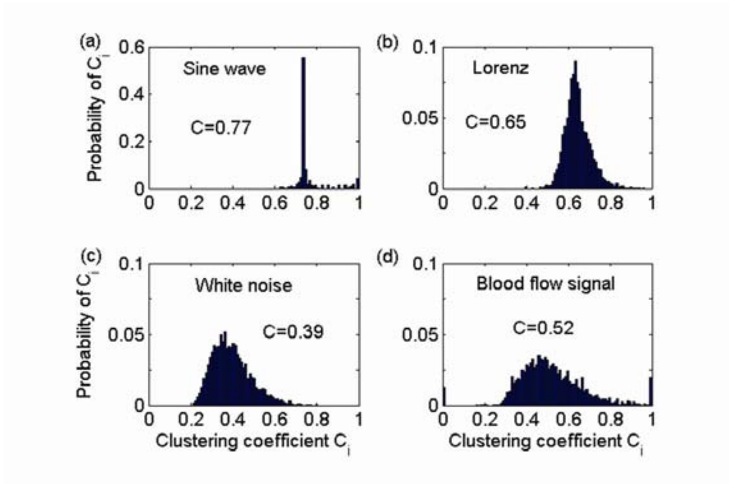

Figure 4. Distributions of C_i values of (a) the sine wave (Figure 1a), (b) the x-component of the Lorenz system (Figure 1b), (c) uniformly distributed white noise (Figure 1c), and (d) the blood flow signal (Figure 1d). The global clustering coefficients C of the four signals are 0.77, 0.65, 0.39, and 0.52, respectively. The selected value of ε corresponds to a global recurrence rate of RR=0.03.

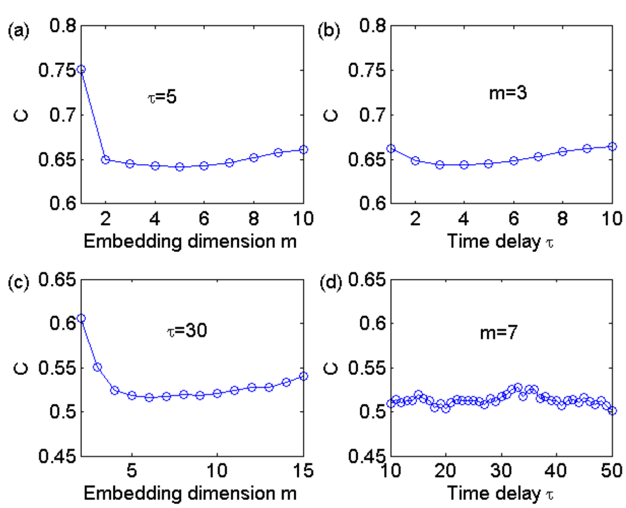

Figure 5. Influence of time delay τ and embedding dimension m on the global clustering coefficient C of (a)(b) the Lorenz system (equation 3) and (c)(d) the blood flow signal (Figure 1d). For the Lorenz system, the optimal values of τ and m are 5 and 3 respectively; for the blood flow signal, the optimal values of τ and m are 30 and 7, respectively. The selected value of ε corresponds to a global recurrence rate of RR=0.03.

Figure 4 shows the probability distributions of C_i values of the four signals shown in Figure 1. For the sine wave, most values of C_i are concentrated around 0.77. Such high and concentrated values are due to the deterministic behavior of the phase space trajectory, on which a large number of neighbors of a specific state are also neighbored. The values of C_i of the Lorenz system are lower and more scattered than those of the sine wave, but higher and more concentrated than those of white noise. C_i values of the blood flow signal are higher than those of white noise and their distribution shape is similar to that of white noise. Based on these considerations, we argue that a higher value of global clustering coefficient indicates a higher degree of determinism of the dynamics and the distribution of local clustering coefficients may be an indicator of the complexity of the dynamics.

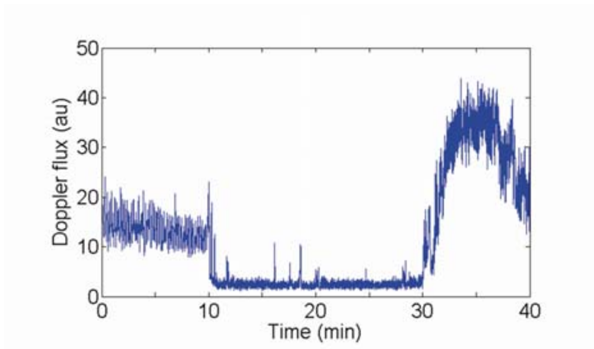

Figure 6. Skin blood flow response to pressure loading in a typical subject

The measure C involves three parameters: time delay τ, embedding dimension m, and the distance threshold ε (determined by the global recurrence rate RR). C is expected to monotonically increases with RR. To investigate the influences of time delay τ and embedding dimension m on C, we calculated C for the x-component of the Lorenz system and the skin blood flow signal (see Figure 1b, d) for wide ranges of τ and m around their optimal values (same as those shown in Figure 3). The results showed that the measure C is robust to the embedding parameters (see Figure 5).

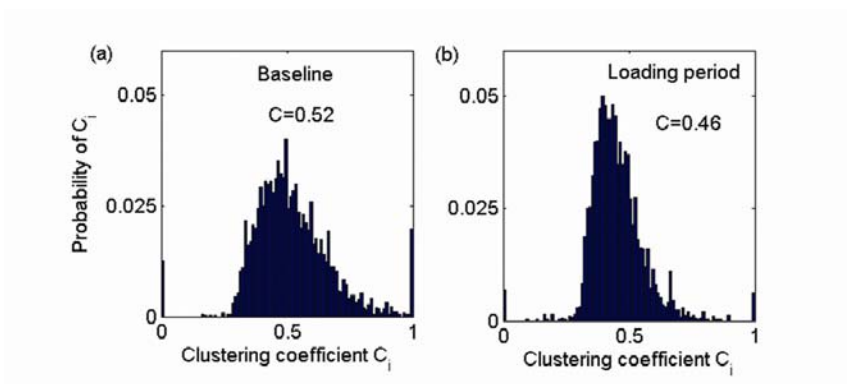

Figure 7. Probability distributions of C_i of two segments of skin blood flow signal shown in Figure 6. (a) A segment of baseline. (b) A segment of loading period.

4 Application to blood flow signals

4.1 Data acquisition

We recruited 10 healthy, young subjects (4 male and 6 female) for this study. Their demographic data were: age 23.7±2.4 (mean ± standard deviation) years; height 1.69±0.09 m; and weight 59.8±6.3 kg. The exclusion criteria included the presence of pressure ulcers on the sacrum, diabetes, vascular diseases, hypertension, or use of vasoactive medicines. This study was approved by the University Institutional Review Board. Informed consent was obtained from each subject before any testing. After at least a 30 min quiet rest period to become acclimated to the room temperature (24±2˚C), the subject was positioned in a prone posture. Sacral skin blood flow was recorded using a laser Doppler flowmetry (PF 5001, Perimed AB, Sweden) at a sampling frequency of 32 Hz. An indenter was used to apply 60 mmHg loading to the sacral skin through the probe. The protocol included a 10 min baseline, a 20 min loading and a 10 min recovery period. Figure 6 shows skin blood flow response to applied pressure in a typical subject.

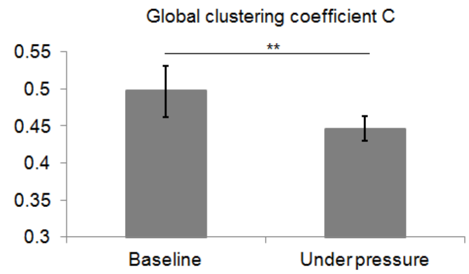

Figure 8. Comparisons of global clustering coefficient C of BFO between baseline and loading period. Values are means ± standard deviations. ***p<0.01

4.2 Data analysis

We calculated local clustering coefficient C_i and then global clustering coefficient C for the blood flow data at baseline and during loading. First, the embedding parameters, time delay τ and embedding dimension m, were separately selected for each data series. Then, to avoid the possible influence of non-stationary, the data series was subdivided into non-overlapping epochs consisting of 4000+$(m-1)\tau$ samples, i.e. 4000 phase space vectors. Given the sampling rate of 32 Hz and assuming that the lower frequency bound of the characteristic frequency associated with endothelial-related metabolic activity is 0.0095 Hz [6], such an epoch contains at least one cycle of the metabolic frequency. This means that such an epoch preserves all power of the five characteristic components embedded in skin BFO. Since C increases with RR, we calculate C of each epoch for RR=0.01~0.08. The obtained C values were averaged over all the epochs.

To investigate whether a decrease in C due to loading is associated with a loss of nonlinearity of BFO, we performed the following tests using surrogate time series. For each original data series, 30 surrogate series were generated using the iterative amplitude adjusted Fourier transform approach [26]. This approach eliminates nonlinearity, preserving the linear features of the original time series, e.g. power spectrum and amplitude distribution [26]. Then we calculated C for the surrogate series for a range of RR=0.01~0.08.

4.3 Statistical analysis

We compared C between baseline and during loading using a paired t-test (the differences of C between the two conditions were normally distributed). One sample t-tests were used to compare C of an original data series and those of surrogate series.

5 Results

Figure 7 shows the probability distributions of local clustering coefficients C_i of blood flow signal at baseline and during loading period (see Figure 6). Compared with the baseline, distribution of C_i during loading period was more concentrated and the global clustering coefficient C was lower. Figure 8 shows the statistical results of C in the ten subjects. C of BFO significantly decreased in response to loading (p<0.01). Surrogate tests indicated that C of BFO at baseline was significantly larger than that of surrogate data (p<10^4), whereas C of BFO during loading was almost equal to that of surrogate data (p>0.05) (see Figure 9).

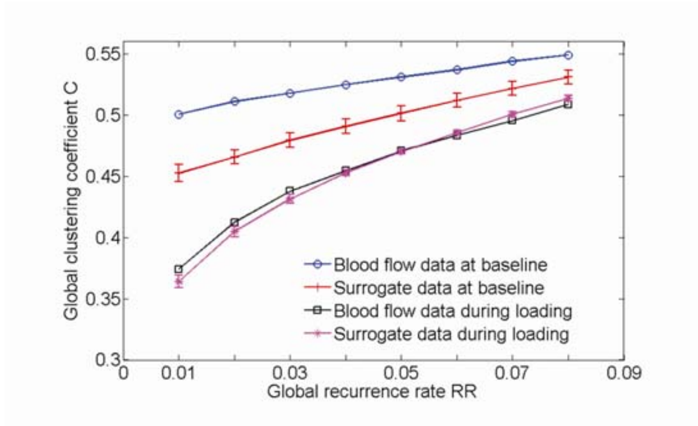

Figure 9. An example of results of surrogate tests. For each original data set, 30 surrogate series were generated. The centers of the error bars represent the mean values of C for 30 surrogate series, and the lengths of the error bars represent the standard deviations.

6 Conclusion

Recurrence is a fundamental property of dynamical processes. The recurrence network approach combines two successful concepts in the study of dynamical and complex systems. In this approach, the recurrence matrix of a time series is interpreted as an adjacent matrix of a network, and measures of topological properties of complex networks are employed to characterize the dynamics of the studied system. Since some measures of complex networks, e.g. local clustering coefficient, characterize higher-order properties of the phase space, this approach provides a complementary view for the study of time series. Our simulation study showed that time series with different types of dynamics exhibit distinct global clustering coefficients and distributions of local clustering coefficients. Additionally, the global clustering coefficient is robust to the embedding parameters. By applying this approach, we observed an altered dynamic property of skin BFO in response to pressure loading, which was associated with a loss of nonlinearity of BFO.

One limitation of this approach is the loss of information about temporal structures. In particular, temporal correlations between individual data points are not taken into consideration. This makes the recurrence network approach distinctively different from many other methods of time series analysis and is an important problem for the analysis of many real-world time series. Nevertheless, as we have mentioned, this approach provides new quantitative characteristics related to the dynamical complexity of a time series, which are not yet provided by the existing methods of time series analysis. Therefore, we argue that this approach can serve as a complementary method of time series analysis. Moreover, because the recurrence network approach does not require long data series, we are able to observe changes in the dynamics with time by applying a sliding window to the data series. In summary, our study suggests that recurrence network approach is a promising tool for analyzing the nonlinear dynamics of BFO.

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