The Impact of Free Agency on Players' Compensation

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Abstract

This study examines whether players benefit from free agency or not. In professional sports, free agency has been a hot topic in recent years. Based on a bargaining model with arbitrary bargaining power, we found that players who have a relative smaller bargaining power to their owners still get lower payoffs after the implement of free agency, and vice versa. Moreover, the expected payoffs of players and sports teams are both not influenced by free agency.

Keywords: free agency, Nash bargaining, Shapley value

1. Introduction

Rottenberg (1956) is the first one who uses economic analysis to discuss the impact of transfer restrictions and free agency on professional sports. He finds that players benefit from free agency to get reasonable salaries, but players get lower salary than their marginal productivity under transfer restrictions. (Note 1) However, Feess and Muehlheusser (2003) show that sports teams lose their wants to nurture talents because of free agency.

Free agency sounds good to players, however, some scholars have different thoughts. Dietl et al. (2008) point out that players loss the profits which come from transfer restrictions, so they use a new analysis method – risk allocation. Because of uncertainty and short athletic career, players prefer cash to the possibility of future high payment. However, sports teams don't sign any insured contract because they don't know players' future performances. Only transfer restrictions can make sports teams pay higher. In other words, free agency destroys players' benefits.

This paper is based on Dietl et al. (2008), (Note 2) but we don't consider the situation of risk-averse. Besides, we don't assume that players have the same bargaining power to sports teams because in reality different sports teams and players have different bargaining power. Therefore, we put stress on bargaining power. It means sports teams get higher revenue if they have bigger bargaining power and vice versa. The remainder of this paper is organized as follows. Section 2 is basic model and Section 3 is the analyses of the transfer with transfer restrictions and free agency. Concluding remarks are presented in Section 4.

2. Basic Model

In this paper, we discuss how free agency makes influence to players and baseball teams. We apply game theory model established by Dietl et al. (2008). Let P denote player and S and L symbolize two sports teams – S is a small team and L is a large team.

We assume that there are only two periods, (Note 3) in period 1 player P can sign a contract with S or L and then in the end of period 1 P become a free agent. To avoid moral hazard, we assume that players' performances are random exogenous variables. S_t and L_t mean players' performances in team S and team L ($t \in \{1,2\}$). Besides, player's utility comes from his/her salary and sports teams' utility come from the sum of their players' performances deduct the salary they pay. In the begin of every period, players make a bargain with sports teams which is based on their past performances. We have to know when players first come into professional sports market, their abilities e_0 are well known to all sports teams. That's because every sports team has scout; therefore, $S_0 = L_0 = e_0, e_0 > 0$.

We assume that player has probabilities q and $1-q$, $q \in (0,1)$ to act well or badly. Besides, one sports team is smaller and the other one is bigger, so we differentiate $q > 1/2$ talented players from $q \le 1/2$ untalented players. In order to simplify calculation, we assume players performances become better or worse are the same amount. s and l are players performances in team S and team L and they are all bigger than 0. Based on Dietl et al. (2008) assumption, we assume that $s < l$. To sum up, in period 1 players has q probability to be $e_0 + s$ or $e_0 + l$, on the other hand, is $e_0 - s$ or $e_0 - l$.

Now we use player P and team S as the sample, the branch chart is:

Using Figure 1, we can calculate player P's performances in every period. When $t = 0$ and in period 1, player P in team S and team L's performances are:

$$E_0[S_1] = e_0 + s(2q-1) \text{ and } E_0[L_1] = e_0 + l(2q-1).$$

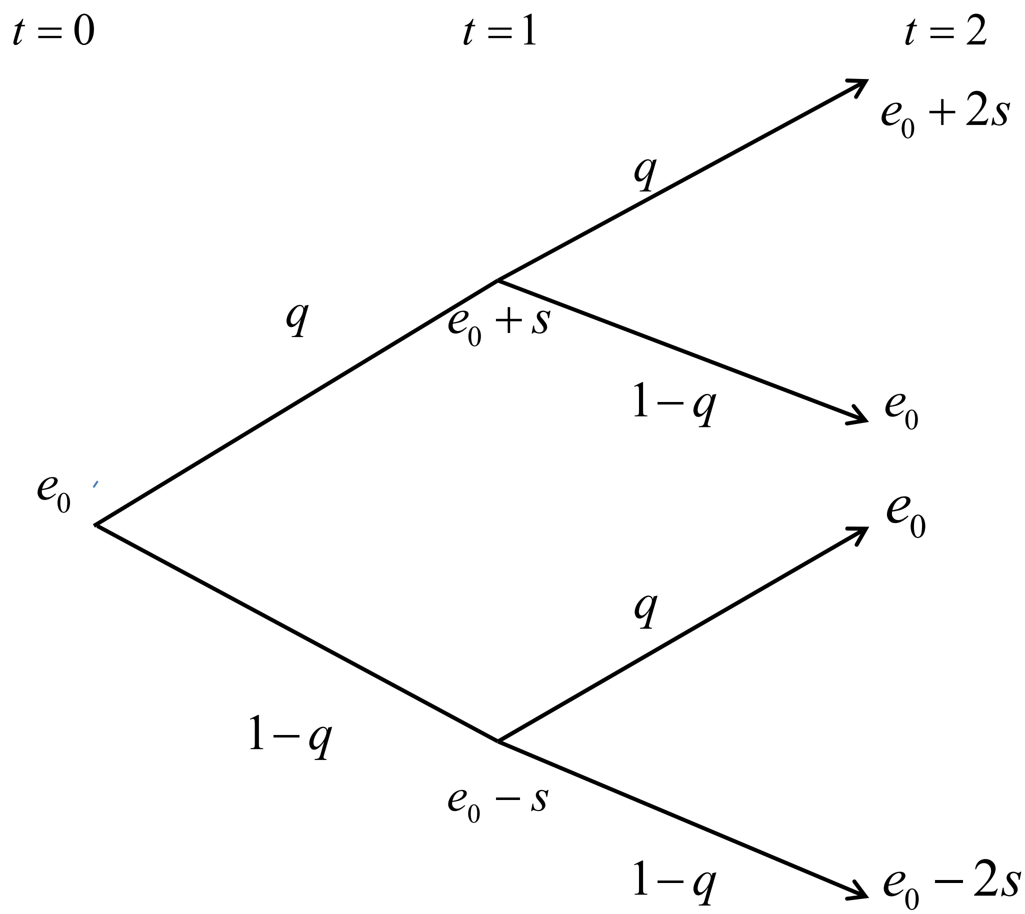

Figure 1. Player P in team S's branch chart

Besides, when $t = 0$ and in period 2, player P in team S and team L's performances are:

$$E_0[S_2] = e_0 + 2s(2q-1) \text{ and } E_0[L_2] = e_0 + 2l(2q-1).$$

We find that when $q \le 1/2$, $E_0[S_1] > E_0[L_1]$ and $E_0[S_2] > E_0[L_2]$, for players who act badly, signing contract with small team is the better choice. On the contrary, for players who act well, signing contract with big team is the better choice. Besides, players who have good performances in period 1, (Note 4) their performances in period 2 are:

$$E_1[S_2^+] = e_0 + 2sq \text{ and } E_1[L_2^+] = e_0 + 2lq.$$

Players who have bad performances in period 1, their performances in period 2 are:

$$E_1[S_2^-] = e_0 + 2s(q-1) \text{ and } E_1[L_2^-] = e_0 + 2l(q-1).$$

No matter q equals what, we get the results $E_1[S_2^+] < E_1[L_2^+]$ and $E_1[S_2^-] > E_1[L_2^-]$. Therefore, we know that players who perform well in period 1 should stay in big team in period 2 and vice versa.

Based on former assumption, next chapter we discuss two situations of transfer: in section 3.1 is under transfer restrictions and in section 3.2 is under free agency.

3. Comparisons between Transfer Restrictions and Free Agency

3.1 Transfer Restrictions

We use Nash bargaining as the model of bargaining between players and sports teams. In period 1, players and sports teams use threat point to negotiate. Besides, all people know the rule that is when the contracts terminate all players' need their original teams' permission to transfer to the other team.

Using Nash bargaining, we have to know players and sports teams' total expected payoffs. In period 1 player P may sign contract with team S or team L, so we discuss two conditions separately.

Condition 1: Signing contract with team S in period 1.

We define $\underline{w}_{r,1}^{S}$ as in period 1 players' salary in team S. (Note 5) r means there is transfer restrictions and lower bar means that players sign contract with team S. On the other hand, upper bar means players sign contract with team L.

When player P acts well in period 1, according to chapter 2, he should make contract with team L, but in this condition, player P, the original team and the new team become a cooperative game. Therefore, we use Shapley value to solve the problem. The formula is:

$$\sum_{C|i\in C}\frac{(c-1)!(n-c)!}{n!}\left[v(C)-v(C-\{i\})\right] \tag{1}$$

i means the player i, c means the number of players in league C, n is the sum of players in this game, $v(C)$ means the total payoffs in league C and $v\left(C-\{i\}\right)$ means the total payoffs that player i can get before he leaves the league C.

Using Shapley value, we can calculate the total payoffs that player P can get. Therefore, in period 2 player P in team L can get payoffs: (Note 6)

$$\underline{w}_{r,2}^{L+}=\frac{1}{6}E_1\left[S_2^+\right]+\frac{1}{3}E_1\left[L_2^+\right].$$

In period 2, expected payoffs that team S can get is the transfer fee T^S:

$$T^S=E_1\left[\pi_{r,2}^{S+}\right]=\frac{1}{6}E_1\left[S_2^+\right]+\frac{1}{3}E_1\left[L_2^+\right].$$

In period 2, team L's expected payoffs are:

$$E_1\left[\pi_{r,2}^{L+}\right]=\frac{1}{3}\left(E_1\left[L_2^+\right]-E_1\left[S_2^+\right]\right). \text{ (Note 7)}$$

On the other hand, when player P acts well in period 1, he will still stay at team S. Therefore, we'd better use Nash bargaining to discuss player P's salary $\underline{w}_{r,2}^{S-}$ in period 2.

The expected utility that team S can get in period 2 is $E_1\left[S_2^-\right]-\underline{w}_{r,2}^{S-}$ and the utility that player P can get is $\underline{w}_{r,2}^{S-}$. Under transfer restrictions, if salary bargaining fails, player P and team S don't get any utility; therefore, their threat points are all 0. Besides, we use parameter β and $1-\beta$, $\beta\in(0,1)$ represent the relative bargaining power of sports teams and player P.

To sum up, we can get:

$$\underline{w}_{r,2}^{S-} = \operatorname{argmax}\left\{\left(E_1\left[S_2^-\right] - \underline{w}_{r,2}^{S-} - 0\right)^{\beta}\left(\underline{w}_{r,2}^{S-} - 0\right)^{1-\beta}\right\} = (1-\beta)E_1\left[S_2^-\right]. \tag{2}$$

That is, player P can get the salary $\underline{w}_{r,2}^{S-} = (1-\beta)E_1\left[S_2^-\right]$ when he stay at team S and team S can get the profits $E_1\left[\pi_{r,2}^{S-}\right] = E_1\left[S_2^-\right] - \underline{w}_{r,2}^{S-} = \beta E_1\left[S_2^-\right]$. However, team L will get nothing, so $E_1\left[\pi_{r,2}^{L-}\right] = 0$.

To here, we can calculate the player P and sports teams' total utility.

The total expected utility of team S is the sum of profits in period 1 $E_0\left[\pi_{r,1}^{S}\right]$ and the profits in period 2 $qE_1\left[\pi_{r,2}^{S+}\right] + (1-q)E_1\left[\pi_{r,2}^{S-}\right]$:

$$\begin{aligned} E_0\left[u_r^S\right] &= E_0\left[\pi_{r,1}^{S}\right] + qE_1\left[\pi_{r,2}^{S+}\right] + (1-q)E_1\left[\pi_{r,2}^{S-}\right] \\ &= \left(E_0\left[S_1\right] - \underline{w}_{r,1}^{S}\right) + q\left(\frac{1}{6}E_1\left[S_2^+\right] + \frac{1}{3}E_1\left[L_2^+\right]\right) + (1-q)\beta E_1\left[S_2^-\right]; \end{aligned} \tag{3}$$

Player P's total expected utility is the sum of salary:

$$\begin{aligned} E_0\left[\underline{u}_r^P\right] &= \underline{w}_{r,1}^{S} + q\underline{w}_{r,2}^{L+} + (1-q)\underline{w}_{r,2}^{S-} \\ &= \underline{w}_{r,1}^{S} + q\left(\frac{1}{6}E_1\left[S_2^+\right] + \frac{1}{3}E_1\left[L_2^+\right]\right) + (1-q)(1-\beta)E_1\left[S_2^-\right]. \end{aligned} \tag{4}$$

Using the information mentioned above, we can use Nash bargaining to calculate unknown $\underline{w}_{r,1}^{S}$ and then we can get threat point that team S has is:

$$d^S = \frac{(1-q)}{3}\left(E_1\left[S_2^-\right] - E_1\left[L_2^-\right]\right),$$

Besides, player P's threat point is $\overline{d}^P$. When he signs contract with team L in period 1, the expected utility $E_0\left[\overline{u}_r^P\right]$ is: (Note 8)

$$\underline{w}_{r,1}^{S} = \operatorname{argmax}\left\{\left(E_0\left[u_r^S\right] - d^S\right)^{\beta}\left(E_0\left[\underline{u}_r^P\right] - \overline{d}^P\right)^{1-\beta}\right\}. \tag{5}$$

To get $E_0\left[\overline{u}_r^P\right]$, we need player P's salary $\overline{w}_{r,1}^{L}$ in period 1 which gave by team L:

$$\overline{w}_{r,1}^{L} = \operatorname{argmax}\left\{\left(E_0\left[u_r^L\right] - d^L\right)^{\beta}\left(E_0\left[\overline{u}_r^P\right] - \underline{d}^P\right)^{1-\beta}\right\}, \text{(Note 9)} \tag{6}$$

Differentiate equations (5) and (6) we can get:

$$\frac{E_0\left[\underline{u}_r^P\right] - \overline{d}^P}{E_0\left[u_r^S\right] - d^S} = \frac{1-\beta}{\beta}; \tag{7}$$

$$\frac{E_0\left[\overline{u}_r^P\right] - \underline{d}^P}{E_0\left[u_r^L\right] - d^L} = \frac{1-\beta}{\beta}. \text{ (Note 10)} \tag{8}$$

Put all information into equations (7) and (8) and then crossover them, we can get unknown variables $\underline{w}_{r,1}^{S}$ and

$\overline{w}_{r,1}^{L}$. Therefore, we can get player P's exactly salary in period 1 is:

$$\begin{aligned}\underline{w}_{r,1}^{S} &= \frac{1}{1+\beta}E_0\left[S_1\right]+\frac{\beta}{1+\beta}E_0\left[L_1\right]+q\frac{1}{6}E_1\left[S_2^+\right]+q\frac{1}{3}E_1\left[L_2^+\right]\\ &\quad+(1-q)\left[\frac{1}{3}(3\beta-1)E_1\left[S_2^-\right]+\frac{1}{3}E_1\left[L_2^-\right]\right]\\ &= \frac{1}{1+\beta}E_0\left[S_1\right]+\frac{\beta}{1+\beta}E_0\left[L_1\right]+qT^S+(1-q)\left(E_1\left[\pi_{r,2}^{S-}\right]-T^L+\frac{1}{2}E_1\left[L_2^-\right]\right)\end{aligned}.$$

Condition 2: Signing contract with team L in period 1.

We use the same way in condition 1 to get unknown variables. When player P transfer to team S in period 2, the expected payoffs of player P, team L and team S are:

$$\overline{w}_{r,2}^{S-}=\frac{1}{6}E_1\left[L_2^-\right]+\frac{1}{3}E_1\left[S_2^-\right];$$

$$T^L=E_1\left[\pi_{r.2}^{L-}\right]=\frac{1}{6}E_1\left[L_2^-\right]+\frac{1}{3}E_1\left[S_2^-\right];$$

$$E_1\left[\pi_{r,2}^{S-}\right]=\frac{1}{3}\left(E_1\left[S_2^-\right]-E_1\left[L_2^-\right]\right).$$

When player P still stay at team L in period 2, the expected payoffs of player P and team L are:

$$\overline{w}_{r,2}^{L+}=(1-\beta)E_1\left[L_2^+\right];$$

$$E_1\left[\pi_{r,2}^{L+}\right]=\beta E_1\left[L_2^+\right].$$

Therefore, under transfer restrictions the total expected utility of team L and player P are:

$$E_0\left[u_r^L\right]=\left(E_0\left[L_1\right]-\overline{w}_{r,1}^{L}\right)+q\beta E_1\left[L_2^+\right]+(1-q)\left(\frac{1}{6}E_1\left[L_2^-\right]+\frac{1}{3}E_1\left[S_2^-\right]\right);$$

$$E_0\left[\overline{u}_r^P\right]=\overline{w}_{r,1}^{L}+q(1-\beta)E_1\left[L_2^+\right]+(1-q)\left(\frac{1}{6}E_1\left[L_2^-\right]+\frac{1}{3}E_1\left[S_2^-\right]\right).$$

We can get the unknown variable $\overline{w}_{r,1}^{L}$ is:

$$\begin{aligned}\overline{w}_{r,1}^{L} &= \frac{1}{1+\beta}E_0\left[L_1\right]+\frac{\beta}{1+\beta}E_0\left[S_1\right]+(1-q)\left(\frac{1}{6}E_1\left[L_2^-\right]+\frac{1}{3}E_1\left[S_2^-\right]\right)\\ &\quad+q\left(\frac{1}{3}(3\beta-1)E_1\left[L_2^+\right]+\frac{1}{3}E_1\left[S_2^+\right]\right)\\ &= \frac{1}{1+\beta}E_0\left[L_1\right]+\frac{\beta}{1+\beta}E_0\left[S_1\right]+(1-q)T^L+q\left(E_1\left[\pi_{r.2}^{L+}\right]-T^S+\frac{1}{2}E_1\left[S_2^+\right]\right)\end{aligned}.$$

3.2 Free Agency

When free agency comes out, we still use Nash bargaining to discuss salary bargaining. But now it's different with section 3.1. player P and the original team both can't make sure that player P will stay in period 2 or not; therefore, the expected utility of Nash bargaining only relates to period 1.

In period 2 we can still use Nash bargaining because the situation is the same with period 1. Therefore, no matter $t=0$ or $t=1$, we all use Nash bargaining to discuss salary bargaining, the formula just as follows:

$$w_{u,t+1}^{S}=\operatorname{argmax}\left\{\left(E_t\left[u_{u,t+1}^{S}\right]-0\right)^{\beta}\left(E_t\left[\underline{u}_{u,t+1}^{P}\right]-E_t\left[\overline{u}_{u,t+1}^{P}\right]\right)^{1-\beta}\right\};\ \text{(Note 11)} \qquad (9)$$

$$w_{u,t+1}^{L}=\text{argmax}\left\{\left(E_t\left[u_{u,t+1}^{L}\right]-0\right)^{\beta}\left(E_t\left[\overline{u}_{u,t+1}^{P}\right]-E_t\left[\underline{u}_{u,t+1}^{P}\right]\right)^{1-\beta}\right\}. \tag{10}$$

No matter team S or L, the expected utility is player P's expected performance deducts the salary. Take team S as example, expected utility is $E_t\left[u_{u,t+1}^{S}\right]=E_t\left[S_{t+1}\right]-w_{u,t+1}^{S}$. To team L the expected utility $E_t\left[u_{u,t+1}^{L}\right]$ is $E_t\left[L_{t+1}\right]-w_{u,t+1}^{L}$. Besides, if sports teams can't sign contract with any player, they will get nothing, so the threat point is 0.

On the other hand, the expected utility of player P is the salary he gets. Therefore, if the bargaining fails, player P still can sign contract with the other team, so his/her threat point is the salary that he can get from the other team $E_t\left[\overline{u}_{u,t+1}^{P}\right]$ is $w_{u,t+1}^{L}$ under the condition $E_t\left[\underline{u}_{u,t+1}^{P}\right]=w_{u,t+1}^{S}$.

Put all information into equation (9) and (10), we can get that player P's salary in team S and team L are:

$$w_{u,t+1}^{S}=\frac{1}{1+\beta}E_t\left[S_{t+1}\right]+\frac{\beta}{1+\beta}E_t\left[L_{t+1}\right]; \tag{11}$$

$$w_{u,t+1}^{L}=\frac{\beta}{1+\beta}E_t\left[S_{t+1}\right]+\frac{1}{1+\beta}E_t\left[L_{t+1}\right]. \tag{12}$$

Under the conditions we mentioned above, we discuss two situations of different contract signing.

Condition 1: Signing contract with team S in period 1.

Using equation (11), we get the player P's salary in period 1:

$$\underline{w}_{u,1}^{S}=\frac{1}{1+\beta}E_0\left[S_1\right]+\frac{\beta}{1+\beta}E_0\left[L_1\right].$$

According to player P's performance in period 1, he has probability q to act well and then he will transfer to team L. P will get salary $w_{u,2}^{L+}$ in period 2:

$$w_{u,2}^{L+}=\frac{\beta}{1+\beta}E_1\left[S_2^{+}\right]+\frac{1}{1+\beta}E_1\left[L_2^{+}\right],$$

Now, because of free agency, team S will get nothing and team L's expected payoffs are:

$$E_1\left[\pi_{u,2}^{L+}\right]=E_1\left[L_2^{+}\right]-\left(\frac{\beta}{1+\beta}E_1\left[S_2^{+}\right]+\frac{1}{1+\beta}E_1\left[L_2^{+}\right]\right).$$

On the other hand, player P's performance in period 1 has probability $1-q$ to act badly and then he will stay at team S. In this situation, profits of player P and team S are:

$$w_{u,2}^{S-}=\frac{1}{1+\beta}E_1\left[S_2^{-}\right]+\frac{\beta}{1+\beta}E_1\left[L_2^{-}\right];$$

$$E_1\left[\pi_{u,2}^{S-}\right]=E_1\left[S_2^{-}\right]-\left(\frac{1}{1+\beta}E_1\left[S_2^{-}\right]+\frac{\beta}{1+\beta}E_1\left[L_2^{-}\right]\right).$$

To sum up, under condition 1, the total expected utility of team S is expected payoffs $E_0\left[S_1\right]-\underline{w}_{u,1}^{S}$ in period 1 pluses the profits $E_1\left[\pi_{u,2}^{S-}\right]$ in period 2 if player P stay at team S. For player P, the expected utility is the salary $\underline{w}_{u,1}^{S}$ in period 1 pluses salary which he might get in period 2. They are:

$$\begin{aligned} E_0\left[u_u^S\right] &= E_0\left[S_1\right]-\underline{w}_{u,1}^{S}+(1-q)E_1\left[\pi_{u,2}^{S-}\right] \\ &= E_0\left[S_1\right]-\underline{w}_{u,1}^{S}+(1-q)\left[E_1\left[S_2^-\right]-\left(\frac{1}{1+\beta}E_1\left[S_2^-\right]+\frac{\beta}{1+\beta}E_1\left[L_2^-\right]\right)\right]; \end{aligned}$$

$$\begin{aligned} E_0\left[\underline{u}_u^P\right] &= \underline{w}_{u,1}^{S}+qw_{u,2}^{L+}+(1-q)w_{u,2}^{S-} \\ &= \underline{w}_{u,1}^{S}+q\left(\frac{\beta}{1+\beta}E_1\left[S_2^+\right]+\frac{1}{1+\beta}E_1\left[L_2^+\right]\right) \\ &\quad +(1-q)\left(\frac{1}{1+\beta}E_1\left[S_2^-\right]+\frac{\beta}{1+\beta}E_1\left[L_2^-\right]\right). \end{aligned}$$

Condition 2: Signing contract with team L in period 1.

Using equation (12), we get the player P's salary $\overline{w}_{u,1}^{L}$ in period 1: $\overline{w}_{u,1}^{L}=\frac{\beta}{1+\beta}E_0\left[S_1\right]+\frac{1}{1+\beta}E_0\left[L_1\right]$.

When player P transfers to team S, the expected payoffs of player P, team L and team S are:

$$w_{u,2}^{S-}=\frac{1}{1+\beta}E_1\left[S_2^-\right]+\frac{\beta}{1+\beta}E_1\left[L_2^-\right],$$

$$E_1\left[\pi_{u,2}^{L-}\right]=0,$$

$$E_1\left[\pi_{u,2}^{S-}\right]=E_1\left[S_2^-\right]-\left(\frac{1}{1+\beta}E_1\left[S_2^-\right]+\frac{\beta}{1+\beta}E_1\left[L_2^-\right]\right).$$

When player P still stay at team L, the expected payoffs are:

$$w_{u,2}^{L+}=\frac{\beta}{1+\beta}E_1\left[S_2^+\right]+\frac{1}{1+\beta}E_1\left[L_2^+\right];$$

$$E_1\left[\pi_{u,2}^{L+}\right]=E_1\left[L_2^+\right]-\left(\frac{\beta}{1+\beta}E_1\left[S_2^+\right]+\frac{1}{1+\beta}E_1\left[L_2^+\right]\right).$$

By using the same way with condition 1, we can get the total expected utility of team L and player P are:

$$\begin{aligned} E_0\left[u_u^L\right] &= E_0\left[L_1\right]-\overline{w}_{u,1}^{L}+qE_1\left[\pi_{u,2}^{L+}\right] \\ &= E_0\left[L_1\right]-\overline{w}_{u,1}^{L}+q\left[E_1\left[L_2^+\right]-\left(\frac{\beta}{1+\beta}E_1\left[S_2^+\right]+\frac{1}{1+\beta}E_1\left[L_2^+\right]\right)\right]; \end{aligned}$$

$$\begin{aligned} E_0\left[\overline{u}_u^P\right] &= \overline{w}_{u,1}^L + q w_{u,2}^{L+} + (1-q) w_{u,2}^{S-} \\ &= \overline{w}_{u,1}^L + q\left(\frac{\beta}{1+\beta}E_1\left[S_2^+\right] + \frac{1}{1+\beta}E_1\left[L_2^+\right]\right) \\ &\quad + (1-q)\left(\frac{1}{1+\beta}E_1\left[S_2^-\right] + \frac{\beta}{1+\beta}E_1\left[L_2^-\right]\right). \end{aligned}$$

3.3 Comprehensive Analysis

Summarize section 3.1 and section 3.2, the results are Table 1 and Table 2:

Table 1. Under transfer restrictions player P signs contract with team S

Condition 1	Period 1	Period 2
Player P	$\frac{1}{1+\beta}E_0\left[S_1\right]+\frac{\beta}{1+\beta}E_0\left[L_1\right]$ $+qT^S+(1-q)\left(E_1\left[\pi_{r.2}^{S-}\right]-T^L+\frac{1}{2}E_1\left[L_2^-\right]\right)$	$q\left(\frac{1}{6}E_1\left[S_2^+\right]+\frac{1}{3}E_1\left[L_2^+\right]\right)$ $+(1-q)(1-\beta)E_1\left[S_2^-\right]$
Team S	$E_0\left[S_1\right]-\left[\frac{1}{1+\beta}E_0\left[S_1\right]+\frac{\beta}{1+\beta}E_0\left[L_1\right]\right.$ $\left.+qT^S+(1-q)\left(E_1\left[\pi_{r.2}^{S-}\right]-T^L+\frac{1}{2}E_1\left[L_2^-\right]\right)\right]$	$q\left(\frac{1}{6}E_1\left[S_2^+\right]+\frac{1}{3}E_1\left[L_2^+\right]\right)$ $+(1-q)\beta E_1\left[S_2^-\right]$
Team L	0	$q\frac{1}{3}\left(E_1\left[L_2^+\right]-E_1\left[S_2^+\right]\right)$

Table 2. Under free agency player P signs contract with team S

Condition1	Period 1	Period 2
Player P	$\frac{1}{1+\beta}E_0\left[S_1\right]+\frac{\beta}{1+\beta}E_0\left[L_1\right]$	$q\left(\frac{\beta}{1+\beta}E_1\left[S_2^+\right]+\frac{1}{1+\beta}E_1\left[L_2^+\right]\right)$ $+(1-q)\left(\frac{1}{1+\beta}E_1\left[S_2^-\right]+\frac{\beta}{1+\beta}E_1\left[L_2^-\right]\right)$
Team S	$E_0\left[S_1\right]-$ $\left(\frac{1}{1+\beta}E_0\left[S_1\right]+\frac{\beta}{1+\beta}E_0\left[L_1\right]\right)$	$(1-q)\left[E_1\left[S_2^-\right]-\left(\frac{1}{1+\beta}E_1\left[S_2^-\right]+\frac{\beta}{1+\beta}E_1\left[L_2^-\right]\right)\right]$
Team L	0	$q\left[E_1\left[L_2^+\right]-\left(\frac{\beta}{1+\beta}E_1\left[S_2^+\right]+\frac{1}{1+\beta}E_1\left[L_2^+\right]\right)\right]$

Using data in Tables 1 and 2, we can get the utility change of player P, team S and team L when free agency comes out:

$$\frac{(2\beta-1)\left[q\left(E_1\left[S_2^+\right]-E_1\left[L_2^+\right]\right)+(1-q)\left(E_1\left[L_2^-\right]-E_1\left[S_2^-\right]\right)\right]}{3(1+\beta)};$$

$$\frac{(2\beta-1)\left[(1-q)E_1\left[S_2^-\right]-(1-q)E_1\left[L_2^-\right]\right]}{3(1+\beta)};$$

$$\frac{(2\beta-1)\left[qE_1\left[L_2^+\right]-qE_1\left[S_2^+\right]\right]}{3(1+\beta)}.$$

Under condition 2, we can summarize as Tables 3 and 4:

Table 3. Under transfer restrictions player P signs contract with team L

Condition 2	Period 1	Period 2
Player P	$\frac{1}{1+\beta}E_0\left[L_1\right]+\frac{\beta}{1+\beta}E_0\left[S_1\right]$ $+(1-q)T^L+q\left(E_1\left[\pi_{r,2}^{L+}\right]-T^S+\frac{1}{2}E_1\left[S_2^+\right]\right)$	$q(1-\beta)E_1\left[L_2^+\right]$ $+(1-q)\left(\frac{1}{6}E_1\left[L_2^-\right]+\frac{1}{3}E_1\left[S_2^-\right]\right.$
Team S	0	$(1-q)\left[\frac{1}{3}\left(E_1\left[S_2^-\right]-E_1\left[L_2^-\right]\right)\right]$
Team L	$E_0\left[L_1\right]-\left[\frac{1}{1+\beta}E_0\left[L_1\right]+\frac{\beta}{1+\beta}E_0\left[S_1\right]\right.$ $\left.+(1-q)T^L+q\left(E_1\left[\pi_{r,2}^{L+}\right]-T^S+\frac{1}{2}E_1\left[S_2^+\right]\right)\right]$	$q\beta E_1\left[L_2^+\right]$ $+(1-q)\left(\frac{1}{6}E_1\left[L_2^-\right]+\frac{1}{3}E_1\left[S_2^-\right]\right)$

Table 4. Under free agency player P signs contract with team L

Condition 2	Period 1	Period 2
Player P	$\frac{1}{1+\beta}E_0\left[L_1\right]+\frac{\beta}{1+\beta}E_0\left[S_1\right]$	$q\left(\frac{\beta}{1+\beta}E_1\left[S_2^+\right]+\frac{1}{1+\beta}E_1\left[L_2^+\right]\right)$ $+(1-q)\left(\frac{1}{1+\beta}E_1\left[S_2^-\right]+\frac{\beta}{1+\beta}E_1\left[L_2^-\right]\right)$
Team S	0	$(1-q)\left[E_1\left[S_2^-\right]-\left(\frac{1}{1+\beta}E_1\left[S_2^-\right]+\frac{\beta}{1+\beta}E_1\left[L_2^-\right]\right)\right]$
Team L	$E_0\left[L_1\right]-\left(\frac{1}{1+\beta}E_0\left[L_1\right]+\frac{\beta}{1+\beta}E_0\left[S_1\right]\right)$	$q\left[E_1\left[L_2^+\right]-\left(\frac{1}{1+\beta}E_1\left[L_2^+\right]+\frac{\beta}{1+\beta}E_1\left[S_2^+\right]\right)\right]$

According to the information in Tables 3 and 4, we can get the utility change of player P, team S and team L when free agency comes out:

$$\frac{(2\beta-1)\left[q\left(E_1\left[S_2^+\right]-E_1\left[L_2^+\right]\right)+(1-q)\left(E_1\left[L_2^-\right]-E_1\left[S_2^-\right]\right)\right]}{3(1+\beta)};$$

$$\frac{(2\beta-1)\left[(1-q)E_1\left[S_2^-\right]-(1-q)E_1\left[L_2^-\right]\right]}{3(1+\beta)};$$

$$\frac{(2\beta-1)\left[qE_1\left[L_2^+\right]-qE_1\left[S_2^+\right]\right]}{3(1+\beta)}.$$

Using the given values above and the information in section 3.2, we can get the results in Table 5:

Table 5. The utility change of player P and two teams when free agency comes out

	Condition 1	Condition 2
Player P	$-\frac{2(2\beta-1)(l-s)\left[q^2+(1-q)^2\right]}{3(1+\beta)}$	$-\frac{2(2\beta-1)(l-s)\left[q^2+(1-q)^2\right]}{3(1+\beta)}$
Team S	$\frac{2(2\beta-1)(l-s)(1-q)^2}{3(1+\beta)}$	$\frac{2(2\beta-1)(l-s)(1-q)^2}{3(1+\beta)}$
Team L	$\frac{2(2\beta-1)(l-s)q^2}{3(1+\beta)}$	$\frac{2(2\beta-1)(l-s)q^2}{3(1+\beta)}$

According to Table 5, because of $l>s$ player P and sports teams who benefits from free agency depending on β which is bargaining power. Therefore, we can get proposition 1, 2 and 3.

[Proposition 1] Bargaining power is the key point in salary bargaining. When $\beta>(<)1/2$, free agency benefit (harm) sports teams and harm (benefit) players; when $\beta=1/2$, there is no influence either to players or sports teams.

When $\beta>1/2$, that is, the bargaining power of the original team is bigger than players, we found that, to team S or L,the utility all bigger than 0 when free agency comes out. In other words, when the sports team is the leader in salary bargaining, even free agency comes out can't lower their profits. On the other hand, when players' bargaining power is smaller than sports teams, players are still followers even under free agency. The result we get just the same as Dietl et al. (2008).

On the contrary, when $\beta<1/2$, players become leaders in salary bargaining. It's the same with Rottenberg (1956) and Muehlheusser (2003).

In proposition 1, we found that bargaining power influences on players and sports teams'utility change under free agency. Therefore, we get the same results with past papers. In reality, the bargaining power of sports teams usually bigger than players, therefore, free agency benefits sports teams rather than players.

[Proposition 2] Free agency is a Zero-Sume Game.

Sum the utility in Table 5, condition 1 and 2, we can get 0, that is, free agency is a Zero-Sum Game. In other words, free agency can't create more value.

[Proposition 3] The utility change doesn't relate to players' abilities.

According to condition 1 and 2, we found that under different contract signing situations we can get the same utility change. In other words, players' abilities and signing contract with whom in period 1 don't influence utility change.

4. Discussion

We focus on the utility change under transfer restrictions and free agency among players, the original team and the other team. After math analysis and the assumption of arbitrary bargaining power, we found that bargaining power is the key point in salary bargaining. Besides, we also found that free agency is a Zero-Sum Game and the utility change doesn't relate to players' abilities. However, we don't take the influences of new contract into account. Maybe this assumption can be discussed by others in the future.

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Notes

Note 1. Scully (1974) shows the same point.

Note 2. Math calculation is based on Burguet et al. (2002) and Feess and Muehlheusser (2002, 2003).

Note 3. $t=0$ to $t=1$ is period 1 and $t=1$ to $t=2$ is period 2.

Note 4. We define $+$ as players have good performances in period 1, and $-$ as players have bad performances in period 1.

Note 5. In transfer restriction, player, the original team and new team's payoff can see appendix 1.

Note 6. When player P cooperate with team S, the payoff is $v(\{P,S\})=E_1\left[S_2^+\right]$. Assume $n=3$ and $c=2$, the probability is $1/6$. Besides, player P can also cooperative with two teams, the payoff is $v(\{P,S,L\})=E_1\left[L_2^+\right]$. Assume $c=3$, the probability is $1/3$. But under transfer restriction, player P cooperates only wirh L, therefore, $v(\{P,L\})=0$.

Note 7. Team L can get the payoff only when cooperative game.

Note 8. We can get the result as follows:

$$\overline{d}^P=E_0\left[\overline{u}_r^P\right]=\overline{w}_{r,1}^L+q\overline{w}_{r,2}^{L+}+(1-q)\overline{w}_{r,2}^{S-}=\overline{w}_{r,1}^L+q(1-\beta)E_1\left[L_2^+\right]+(1-q)\left(\frac{1}{3}E_1\left[S_2^-\right]+\frac{1}{6}E_1\left[L_2^-\right]\right).$$

Note 9. Results as follows:

$$E_0\left[u_r^L\right]=\left(E_0\left[L_1\right]-\overline{w}_{r,1}^L\right)+q\beta E_1\left[L_2^+\right]+(1-q)\left(\frac{1}{6}E_1\left[L_2^-\right]+\frac{1}{3}E_1\left[S_2^-\right]\right),$$

$$d^L=\frac{q}{3}\left(E_1\left[L_2^+\right]-E_1\left[S_2^+\right]\right),$$

$$E_0\left[\overline{u}_r^P\right]=\overline{w}_{r,1}^L+q(1-\beta)E_1\left[L_2^+\right]+(1-q)\left(\frac{1}{6}E_1\left[L_2^-\right]+\frac{1}{3}E_1\left[S_2^-\right]\right).$$

$$E_0\left[\overline{u}_r^P\right] = \overline{w}_{r,1}^L + q(1-\beta)E_1\left[L_2^+\right] + (1-q)\left(\frac{1}{6}E_1\left[L_2^-\right] + \frac{1}{3}E_1\left[S_2^-\right]\right).$$

Note 10. The other threat point is: $\underline{d}^P = \underline{w}_{r,1}^S + q\left(\frac{1}{6}E_1\left[S_2^+\right] + \frac{1}{3}E_1\left[L_2^+\right]\right) + (1-q)(1-\beta)E_1\left[S_2^-\right]$.

Note 11. u means under the free agency.

Appendix

Appendix 1. Under transfer restrictions the payoffs of player P and two teams

Condition 1	Period 1	Period 2 (+)	Period 2 (−)
Player P	$\underline{w}_{r,1}^S$	$\underline{w}_{r.2}^{L+}$ (transfer)	$\underline{w}_{r,2}^{S-}$
Team S	$E_0\left[S_1\right] - \underline{w}_{r,1}^S$	T^S	$E_1\left[\pi_{r.2}^{S-}\right]$
Team L		$E_1\left[\pi_{r.2}^{L+}\right]$	

Appendix 2. Under free agency the payoffs of player P and two teams

Condition 2	Period 1	Period 2 (+)	Period 2 (−)
Player P	$\overline{w}_{r,1}^L$	$\overline{w}_{r.2}^{L+}$	$\overline{w}_{r,2}^{S-}$ (transfer)
Team S			$E_1\left[\pi_{r.2}^{S-}\right]$
Team L	$E_0\left[L_1\right] - \overline{w}_{r,1}^L$	$E_1\left[\pi_{r.2}^{L+}\right]$	T^L